GD2101

OLLSCOIL NA hÉIREANN, GAILLIMH
NATIONAL UNIVERSITY OF IRELAND, GALWAY

SUMMER EXAMINATIONS 2003

FIRST COMMERCE EXAMINATION

MATHEMATICS [MA130]

MA 131 — *CALCULUS*

First Paper

Dr Dave Johnson
Professor T. Hurley
Dr D. L. Flannery
Dr R. Ryan

Time allowed: *three* hours.
Answer *six* questions.

1. (a) Let $y = \dfrac{x-3}{2x+1}$. Find $\lim_{x\to\infty} y$ and $\lim_{x\to-\infty} y$. State the vertical and horizontal asymptotes (if any) of the graph of y.

(b) Evaluate the following limits:

(i) $\displaystyle \lim_{x\to 1} \frac{x^3-2x+1}{x^2-1}$

(ii) $\displaystyle \lim_{x\to\infty} \frac{2x^2+1}{x^2+x-7}$

(iii) $\displaystyle \lim_{x\to 1} \frac{\ln\,(x^2-x+1)}{2x+1}$

(iv) $\displaystyle \lim_{x\to 0} \frac{e^{3x+5}}{x}$

2. (a) Differentiate x^2 from first principles.

(b) Differentiate the following functions with respect to x:

(i) $(x^5-x)^{12}$

(ii) $(x-1)\sqrt{10+x-3x^3}$

(iii) $\dfrac{e^{x^2}}{\ln x}$

(iv) $\ln(e^{x^2+2x+1})$

p.t.o.

3. (a) The monthly demand function for a particular product is

$$q = f(p) = 30,000 - 25p,$$

where q is stated in units and p is stated in euro. Determine the quadratic total revenue function $R = g(p)$. What does the total revenue equal at a price of €60? How many units will be demanded at this price? At what price will total revenue be maximized? What is total revenue at this price?

(b) Let

$$f(x) = \frac{x^4}{4} - \frac{8x^3}{3} + 8x^2.$$

Find $f'(x)$ and the values of x for which $f'(x) = 0$. Find the ranges of values of x for which $f(x)$ is increasing and the ranges of values of x for which $f(x)$ is decreasing. Find the critical points of f and also find the nature of these critical points. Sketch the graph of $y = f(x)$.

4. (a) The total cost of producing q units of a certain product is described by the function

$$C = 12,500,000 + 100q + 0.02q^2.$$

(i) Determine how many units q should be produced in order to minimize the average cost per unit.

(ii) What is the minimum average cost per unit?

(iii) What is the total cost of production at this level of output?

(b) The total cost and revenue functions for a certain product are

$$\begin{aligned} C(q) &= 5,000,000 + 250q + 0.002q^2 \\ R(q) &= 1,250q - 0.005q^2 \end{aligned}$$

Find the marginal cost and marginal revenue. Using the marginal approach, determine the profit-maximizing level of output. What is the maximum profit?

5. (a) Evaluate the following indefinite integrals:

(i) $\int (\frac{2}{x^4} + \sqrt[3]{x} - x^{12})\, dx$, (ii) $\int e^{x^4+1} x^3\, dx$, (iii) $\int (x^2 - 1)\sqrt{x^3 - 3x + 1}\, dx$.

(b) Use integration by parts to evaluate the integral

$$\int x^2 e^{4x+1}\, dx\,.$$

(c) Use the method of partial fractions to evaluate the integral

$$\int \frac{5x+1}{x^2 - x - 2}\, dx\,.$$

p.t.o.

6. (a) Find the general and particular solutions of the following differential equations:

(i) $\dfrac{dy}{dx} = 4x^3 + 9x^2,\ f(-1) = 50.$

(ii) $\dfrac{d^2y}{dx^2} = \dfrac{x^2}{12} + 12x,\ f'(2) = 20,\ f(2) = 40.$

(b) Find the area of the region between the graphs of the functions $f(x) = x^2$ and $g(x) = 2x + 8$ over the interval $1 \leq x \leq 6$.

7. (a) Evaluate the definite integral $\int_2^6 (x^2 - 5)\,dx$.

(b) Find an approximate value for this integral using the Rectangle Rule, the Trapezoidal Rule and Simpson's Rule, subdividing into four intervals in each case.

(c) Which of the approximations found in part (b) is the most accurate?

8. (a) The population of a certain endangered species appears to be declining at an exponential rate. Ten years ago, the population was 5,000. This year, the population is 2,500. Find the exponential decay funtion which describes the population P as a function of time t, and estimate the size of this population one year from now.

(b) Evaluate the following definite integrals:

(i) $\int_0^2 2xe^{x^2}\,dx$,

(ii) $\int_1^3 (x^2 + 9)\,dx + \int_3^1 (10 - 2x^2)\,dx.$