G02102

OLLSCOIL NA hÉIREANN, GAILLIMH

NATIONAL UNIVERSITY OF IRELAND, GALWAY

SUMMER EXAMINATIONS 2003

FIRST COMMERCE EXAMINATION

MATHEMATICS [MA130]

MA 133 — *ALGEBRA*

Second Paper

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Time allowed: *three* hours.
Answer *six* questions.

1. (a) In manufacturing a product, a firm incurs costs of two types. Fixed annual costs of € 250, 000 are incurred regardless of the number of units produced. In addition, each unit produced costs the firm € 6. If C equals total annual cost in euro and x equals the number of units produced during a year:

(i) Determine the annual cost function $C = f(x)$.

(ii) What is $f(200,000)$? What does $f(200,000)$ represent?

(b) The supply and demand functions for a product are $q_s = p^2 - 400$ and $q_d = p^2 - 40p + 2600$. Determine the market equilibrium price and quantity.

2. (a) Use Gaussian Elimination (only) to find the solution to

$$\begin{array}{rcrcrcr} x & + & y & + & z & = & 6 \\ 2x & - & y & + & 3z & = & 4 \\ 4x & + & 5y & - & 10z & = & 13 \end{array}$$

(b) Compute the determinant of

$$D = \begin{pmatrix} 2 & 7 & -2 & 0 \\ 1 & -2 & -3 & 0 \\ 3 & 3 & 6 & 9 \\ 6 & -3 & -2 & 0 \end{pmatrix}.$$

p.t.o.

3. (a) Find the inverse of the following matrix using the cofactor method

$$A = \begin{pmatrix} 1 & 2 & 0 \\ 1 & 0 & -1 \\ -1 & 3 & 2 \end{pmatrix}.$$

(b) Consider the system of equations

$$\begin{array}{rcrcrcl} x & + & 2y & & & = & 1 \\ x & & & - & z & = & 1 \\ -x & + & 3y & + & 2z & = & 1 \end{array}$$

Rewrite this as

$$A \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

where A is the matrix as in part (a). By multiplying through (on the left) by the inverse of A, solve the system of equations.

4. (a) Use the Corner-Point Method to find the maximum and minimum values of the function $z = 3x_1 + 2x_2$ in the region defined by the following constraints:

$$\begin{aligned} x_1 + x_2 &\leq 24 \\ x_1 + 3x_2 &\leq 36 \\ x_2 &\geq 4 \end{aligned}$$

(b) Find the inverse of

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 3 & 0 & 4 \\ 1 & 2 & 5 \end{pmatrix}$$

using *row operations only*. (DO NOT use the cofactor method.)

5. (a) What is the effective annual interest rate of an investment at a nominal rate of 16 percent per year, if interest is compounded (i) semi-annually, (ii) quarterly, (iii) continuously?

(b) If € 2000 is to grow to € 5000 over a 12-year period, at what annual rate of interest must it be invested, given that interest is compounded annually?

p.t.o.

6. (a) A company wishes to establish a sinking fund with the objective of accumulating 600,000 Euro after 8 years. If interest is earned at the rate of 10%, compounded quarterly, determine the size of the quarterly payment required.

   (b) Having won a lottery, you are offered a choice of prize: either 35,000 Euro now, or an annual payment of 10,000 Euro made at the end of this year and each of the following three years. The prevailing interest rate is 4% per year, compounded yearly. Which of these two prizes is more valuable?

7. **Formulate** the following Linear Programming Model. You are **not** required to find the solution.

   A manufacturer mixes three different types of ingredient into three blends of sauce. The three ingredients cost 60, 70 and 90 cents per litre, respectively. The weekly availability of the three ingredients are 100,000, 60,000 and 50,000 litres, respectively. The manufacturer sells the three blends for 5, 8 and 10 Euro per litre, respectively. Total weekly output is required to be at least 130,000 litres. The following blending restrictions must be adhered to:

   (i) Ingredient 1 should constitute at least 30% of blend 3 and at most 20% of blend 1.

   (ii) Ingredient 2 should constitute exactly 15% of blend 3.

   (iii) Ingredient 3 should constitute at least 40% of blend 1 and at least 18% of blend 2.

   The objective is to determine the number of litres of each ingredient that should be used in each blend so as to maximize the total weekly profit.

8. The following table contains the data for a Transportation Problem. The table lists the cost of shipping one unit from each of three origins to each of three destinations. Also listed are the supply capacities of each of the origins and the demands at each destination.

	Destination			
Origin	1	2	3	Supply
1	4	6	2	200
2	6	8	5	600
3	7	3	4	200
Demand	300	250	450	

   (i) Use the Northwest Corner Method to find an initial solution and find the cost of this solution.

   (ii) Use the Stepping Stone Algorithm to find an optimal solution. What is the cost of your solution?