GD2103

OLLSCOIL NA hÉIREANN, GAILLIMH
NATIONAL UNIVERSITY OF IRELAND, GALWAY

SUMMER EXAMINATIONS, 2003

FIRST ENGINEERING & INFORMATION TECHNOLOGY EXAMINATION

MATHEMATICS [MA150]

MA151 — *CALCULUS*

First Paper

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Time allowed: *Three* hours.
Answer six questions.

1. (a) Evaluate two of the following limits.

(i) $\lim_{x\to 2} \frac{x^2-3x+2}{x^2-4}$

(ii) $\lim_{\theta\to\frac{\pi}{2}} \frac{\sin 2\theta}{\frac{\pi}{2}-\theta}$

(iii) $\lim_{x\to\infty} \frac{x^3-3x+4}{2x^3+x^2-x+7}$

(b) Let

$$f(x) = \frac{x^2-4}{x^2+4x+3}$$

(i) Find all the asymptotes of the graph of f.
(ii) Find the intercepts of the graph of f.
(iii Sketch the graph of f.

p.t.o.

2. (a) For which value(s) of k is the following function continuous?

$$g(x) = \begin{cases} 3x+1 & \text{if } x < 1 \\ kx^2 & \text{if } x \geq 1 \end{cases}$$

(b) Using the intermediate value theorem, show that the equation

$$x^3 + x + 11 = 0$$

has a solution.

(c) Using the definition of a limit, show that

$$\lim_{x \to 1} \frac{x^2 - 1}{x - 1} = 2.$$

3. (a) Compute the derivatives of two of the following functions.

(i) $f(x) = x^2 \sin x$

(ii) $g(t) = 2^{3t+1}$

(iii) $h(x) = \dfrac{x^2 + x + 1}{x + 1}$

(b) Using logarithmic differentiation compute the derivative of

$$f(x) = \frac{\sqrt{2x+1}(x^2+1)(x^3+1)^4}{\sqrt[3]{x-1}(x-1)}.$$

(c) Find $\dfrac{dy}{dx}$ if

$$xy^2 - \sin(xy) = 0.$$

4. A water container is to be constructed in the shape of a cylinder with an open top. If $1m^2$ of metal is used to construct the container, what dimensions should it have in order to maximise the volume?

p.t.o.

5. (a) State the Fundamental Theorem of Calculus. Use this theorem to show that

$$\frac{d}{dx}\int_x^{\sec x}\frac{1}{t\sqrt{t^2-1}}dt = 1 - \frac{1}{x\sqrt{x^2-1}}$$

(b) Estimate $\int_1^2 \frac{1}{x}\,dx$ by computing the lower Riemann sum $L(\frac{1}{x},2)$ and the upper Riemann sum $U(\frac{1}{x},2)$ with 2 equal subintervals in each case.
Calculate also $L(\frac{1}{x},4)$ and $U(\frac{1}{x},4)$ and verify that:

$$L(\frac{1}{x},2) < L(\frac{1}{x},4) < \log 2 < U(\frac{1}{x},4) < U(\frac{1}{x},2).$$

6. (a) Evaluate *two* of the following integrals:
(i) $\int \frac{dx}{\sqrt{8x-x^2}}$ (ii) $\int \frac{x+4}{\sqrt{x^2+4}}dx$ (iii) $\int \frac{x^2+2x+2}{x^2+x}dx$

(b) Establish the reduction formula:

$$\int \sec^n x\,dx = \frac{1}{n-1}\sec^{n-2}x\tan x + \frac{(n-2)}{(n-1)}\int \sec^{n-2}x\,dx \qquad (n>1)$$

and use it to evaluate $\int \sec^4 x\,dx$.

7. (a) Find the area of the region in the first quadrant that is bounded by the curve $x = y^2$ and the line $x = y+2$.

(b) Find the length of the curve $y^2 = x^3$ between the points $(0,0)$ and $(4,8)$.

(c) The region bounded by the parabola $y^2 = 4x$ and the line $x = 1$ is rotated about the line $x = -1$. Find the volume of the solid generated.

8. Solve the following differential equations:

(a) $(2y-x)\frac{dy}{dx} = 2x+y;\ \ y(2) = 3$

(b) $x\frac{dy}{dx} - 2y = x^3\cos x$

(c) $\frac{d^2y}{dx^2} + 14\frac{dy}{dx} + 49y = 4e^{5x}$