GX 2109

OLLSCOIL NA hÉIREANN, GAILLIMH
NATIONAL UNIVERSITY OF IRELAND, GALWAY

SUMMER EXAMINATIONS 2003

SECOND UNIVERSITY EXAMINATION

MATHEMATICS
MA203 – LINEAR ALGEBRA

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Time allowed: **Two** hours.
Attempt **three** questions.

1. (a) Find the general solution of the following system of linear equations.

$$\begin{aligned} x + y + z + w &= 1 \\ -x + 2y + 2w &= 1 \\ 2x + 5y + 3z + 4w &= 4 \end{aligned}$$

   (b) Consider the following system of linear equations.

$$\begin{aligned} x - ky + z &= 1 \\ x + y + z &= k \\ 2x + y + kz &= 1 \end{aligned}$$

   where k is some constant. For what values of k does this system have a unique solution?

2. (a) Let $A = \begin{pmatrix} 1 & 2 & 3 \\ t & -1 & 1 \\ -1 & 1 & 2 \end{pmatrix}$

i. For what values of t is the matrix A invertible?

ii. Using the Gauss-Jordan method, compute the inverse of A when $t = -7$.

(b) Let $B = \begin{pmatrix} 1 & 1 & 0 & 1 \\ 2 & 0 & 2 & 2 \\ -1 & 0 & 2 & 1 \\ 2 & 1 & 0 & -1 \end{pmatrix}$. Using the cofactor method, compute the entry in the second row and first column of the matrix B^{-1}.

3. Let $X = \begin{pmatrix} 4 & 3 & -6 \\ 0 & 1 & 0 \\ 3 & 3 & -5 \end{pmatrix}$

(a) Find the eigenvalues of the matrix X.

(b) Write down a matrix P such that $P^{-1}XP$ is a diagonal matrix.

(c) Find a formula for X^k where k is any integer.

4. (a) Let X be the transition matrix of a Markov process.

i. Describe two characteristic properties of X.

ii. Show that 1 is an eigenvalue of X.

(b) A car rental company has offices in Galway, Dublin and Shannon. Each month $\frac{1}{3}$ of the cars rented in Galway are returned in Dublin and $\frac{1}{4}$ of the cars rented in Galway are returned in Shannon - the remainder are returned to Galway. Of the cars rented in Dublin, $\frac{1}{10}$ are returned in Galway, $\frac{1}{5}$ are returned in Shannon and the remainder in Dublin. Finally, of the cars rented in Shannon, $\frac{1}{5}$ are returned in Galway and $\frac{1}{5}$ are returned in Dublin with the remainder being brought back to Shannon. Find the steady state of this Markov process.