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OLLSCOIL NA hÉIREANN, GAILLIMH
THE NATIONAL UNIVERSITY OF IRELAND, GALWAY

SUMMER EXAMINATIONS 2003

SECOND UNIVERSITY (Arts and Science) EXAMINATION

MATHEMATICS [MA212]

PASS

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Professor T. C Hurley,
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Time allowed: *Two* hours.
Full marks for *three* questions.

1. (a) Let $w = \dfrac{xy}{2x+5y}$. Show that

$$2\frac{\partial w}{\partial y} - 5\frac{\partial w}{\partial x} = \frac{2x-5y}{2x+5y}.$$

(b) Let $w = \sqrt{x^2+y^2}$. Evaluate the partial derivatives $\dfrac{\partial w}{\partial x}$, $\dfrac{\partial w}{\partial y}$ and $\dfrac{\partial^2 w}{\partial x \partial y}$ and verify the following identities.

$$\text{(i)}\ \ x\frac{\partial w}{\partial x} + y\frac{\partial w}{\partial y} = w, \quad \text{(ii)}\ \ \left(\frac{\partial w}{\partial x} + \frac{\partial w}{\partial y}\right)^2 + 2w\frac{\partial^2 w}{\partial x \partial y} = 1.$$

2. Let S be the surface

$$3x^2 + 5y^2 + 7z^2 = 15.$$

Prove that the equation of the tangent plane to S at a point (x_0, y_0, z_0) on S is

$$3x_0x + 5y_0y + 7z_0z = 15.$$

Show that the line $x = -1 + 2t$, $y = 2 - t$, $z = 2 - t$ intersects the surface S in two points P and Q and find the coordinates of these points. Find the equations of the tangent planes to S at the points P and Q and find the parametric equations of the line of intersection of these tangent planes.

p.t.o.

3. (a) Find the slope of the surface

$$w = x^3 - xy^2 - xy$$

at the point $(2, 1, 4)$ in the direction $(3, 4)$. Find the direction and the magnitude of the steepest slope to the surface at that point.

(b) Find the stationary points of the function

$$w = x^2y - 3x^2 - 3y^2.$$

and determine if they are local maxima, minima or saddle points.

4. (a) Let $w = f(x, y, z)$ and $x = x(u, v)$, $y = y(u, v)$, $z = z(u, v)$. Express $\frac{\partial w}{\partial u}$, $\frac{\partial w}{\partial v}$ in terms of $\frac{\partial w}{\partial x}$, $\frac{\partial w}{\partial y}$, $\frac{\partial w}{\partial z}$ and $\frac{\partial x}{\partial u}$, $\frac{\partial x}{\partial v}$, $\frac{\partial y}{\partial u}$, $\frac{\partial y}{\partial v}$, $\frac{\partial z}{\partial u}$, $\frac{\partial z}{\partial v}$.

Let $w = x^3 + y^3 + 6z$ and $x = u + v$, $y = u - v$, $z = uv$. Show that

$$\frac{\partial w}{\partial u} + \frac{\partial w}{\partial v} = 6(u + v)(u + v + 1).$$

(b) Find the maximum and minimum values of $x^2 + y^2$, subject to the condition

$$8x^2 + 12xy + 17y^2 = 50.$$