

OLLSCOIL NA hÉIREANN, GAILLIMH
NATIONAL UNIVERSITY OF IRELAND, GALWAY

SUMMER EXAMINATIONS, 2003

MATHEMATICS [MA300]
MA302 — Complex Variables

PASS

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Time allowed: Two hours.

Third Arts Mathematical Studies: Full marks for *four* questions.

All other students: Full marks for *three* questions.

1.

(a) (i) Find $\lim_{z \rightarrow 1+i} \frac{z^2 - 2i}{z^2 - 2z + 2}$.

(ii) Differentiate $\cosh((iz^2 + 3z)^{-4})$.

(b) For $z = x + iy$, let $f(z) = (x^3 + 3xy^2) + i(y^3 + 3x^2y)$. Determine where $f'(z)$ exists and find its value there.

(c) Show that $u(x, y) = x^3 - 3xy^2$ is harmonic and find a harmonic conjugate v of u . Can you write $u + iv$ as a function of the complex variable $z = x + iy$?

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2.

- (a) Solve the equation $z^6 + 1 = \sqrt{3}i$ and locate all solutions graphically.
- (b) Show that $\cos^{-1} z = -i \log(z + i(1 - z^2)^{1/2})$ and find all values of $\cos^{-1}(3i)$.
- (c) Find all values of $(1 - i)^{12i}$.

3.

- (a) Evaluate the complex integrals

$$\int_C \operatorname{Re}(z) dz \quad \text{and} \quad \int_C (9z^2 - 6iz) dz,$$

where C is a contour joining the point $-i$ to the point i along

- (i) the polygonal path with vertices $-i$, 1 , and i ;
 - (ii) the right semicircle $\{z : |z| = 1, \operatorname{Re}(z) \geq 0\}$.
- (b) State Cauchy's integral formula for derivatives, and use it to evaluate

$$\int_C \frac{e^{z^2}}{(z - i)^4} dz,$$

where C is the positively oriented circle $|z| = 3$.

4.

- (a) Find the Laurent series about $z_0 = 0$ for

$$f(z) = \frac{6z + 10}{z^2 + 2z - 3}$$

valid in (i) $|z| < 1$; (ii) $1 < |z| < 3$.

- (b) State the Residue Theorem, and use it to evaluate the integral

$$\int_C \frac{1}{z^4 + z^3 - 2z^2} dz,$$

where C is the positively oriented circle about centre 0 with radius 3 .