GX2119

OLLSCOIL NA hÉIREANN, GAILLIMH
NATIONAL UNIVERSITY OF IRELAND, GALWAY

SUMMER EXAMINATIONS, 2003

THIRD UNIVERSITY EXAMINATION

MA310 - ACTUARIAL MATHEMATICS 1

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Dr. M. Hayes

Time allowed: *Two* hours.

Marks shown for each question.

In addition to this paper you should have available actuarial tables and an electronic calculator which is not capable of storing text.

PART A: ANSWER TWO OUT OF THREE QUESTIONS.

1. If the nominal rate of interest convertible every 4 months is 4% p.a., calculate the nominal rate of interest p.a. convertible every 6 months and the nominal rate of discount p.a. convertible every 2 months.

[5]

2. Show that

$$\ddot{s}^{(p)}_{n|} = \frac{(1+i)^n - 1}{d^{(p)}}.$$

[5]

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3. Find the present value of a continuously payable annuity $\rho(t)$ where

$$\rho(t) = \frac{1}{2}t + 2$$

over the next 10 years if the force of interest is $\frac{1}{2}\%$ p.a.

[5]

PART B: ANSWER TWO OUT OF THREE QUESTIONS.

4. A project has an initial outlay on 1/6/2003 of £1,000,000. Six months later a further expenditure of £500,000 will be required. On 1/6/2004 income will be received of £12,000 a month payable in advance for 12 years. The income increases by 5% per annum compound on 1st June each year starting in 2009.

Calculate the net present value of the project at a rate of 6% per annum.

[10]

pto

5. The business plan for a taxi company that has obtained a 25 year lease is shown below:

Cashflow	**Timing**	**Amount (£'000's)**
Initial Set Up Costs	Start	-200
Advertising Income	4 months	20
Government Grant	5 months	10
One-off payment to cab sales company to provide cabs over 25 years	5 months	- 1000
Client Payments	Continuous from 5 months	200 p.a.
Staff Costs	Continuous from 5 months	-25 p.a.
Resale Value of Assets	25 years	10

Calculate the discounted payback period for the project assuming it will be financed by a flexible loan facility based on an effective return of 6% p.a.

[10]

pto

6. The following are total valuations of an insurance fund (£000's).

Year	1st Jan	1st April	1st July	1st October
2001	759	801	799	602
2002	702	659	640	800
2003	900			

On 31st March, 2001, £60,000 worth of fixed interest securities were sold. On 30th June, 2001 equity dividends of £20,000 were received together with a company contribution of £50,000. Interest on cash of £15,000 was received on 30th September, 2002. Investment managers were paid £20,000 from the fund on 15th November, 2002.

(a) Calculate the money weighted rate of return p.a. in each quarter of 2001 and 2002.

(b) Calculate the quarterly linked internal rate of return p.a. for the years 2001 and 2002.

[10]

PART C: ANSWER TWO OUT OF THREE QUESTIONS.

7. A fixed interest stock pay a coupon of 8% per annum every three months, the next coupon is due on 1st June, 2003. It is redeemable at 115% at the option of the borrower on any 1st December between 2017 and 2020 inclusive.

Calculate the price an investor would pay on 31st May, 2003 to receive an effective net yield of at least 10% per annum. The investor pays tax at 32% on 31st December on income received during the previous 12 months and capital gains tax at 30% 2 years following realisation of any gain.

[17.5]

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8. An insurance company who is taxed at 40%, purchased 1000 shares on 30th June, 2002, in a company ex dividend. All income tax on income received in a calendar year is paid on 31st December of that year. Dividends are paid twice yearly and the last dividend of £2 per share was paid 3 months before date of purchase. The dividends are expected to rise by 4% each calendar year. Calculate the price paid by the investor if the expected net yield is 13% p.a. effective.

[17.5]

9. A two year index-linked security will be issued on 1st July, 2003. It pays nominal coupons of 4% annually in arrear and is redeemed at par. The coupons and capital payment are inflated by reference to the inflation index 12 months before the payment is made. The inflation index on 1st July, 2002 was 579 and the table below shows the index values at other times.

Time	**1/7/03**	**1/7/04**	**1/7/05**
Index	599	602	670

(a) Calculate the average annual nominal rate of return obtained by the investor if the price of the stock is £100 per £100 nominal.

(b) Calcuate the average annual real rate of return obtained by the investor.

(c) Explain the difference between (a) and (b).

[17.5]

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PART D: ANSWER ONE OUT OF ONE QUESTION.

10. (a) Show that the forward price K for a security with fixed cash income is

$$K = S_0 e^{\delta T} - C e^{\delta(T-t_1)}$$

where S_0 is the current price of the security, δ is the force of interest, C is the cash income due at time t_1, and T is the time when the contract will mature.

(b) A fixed interest security pays annual coupons of 9% which are payable half-yearly in arrear and is redeemable at 110%. Two months before the next coupon is due, an investor negotiates a forward contract in which he agrees to buy £20,000 nominal in 5 months time. The current price of the stock is £90 per £100 nominal and the effective rate of interest is 4% p.a. Calculate the forward price.

[15]

PART E: ANSWER TWO OUT OF THREE QUESTIONS.

11. (a) Calculate the price of a five year fixed interest security redeemable at par with 8% annual coupons given the following information:

$$\begin{aligned} y_1 &= 3\% \\ y_2 &= 4\% \\ f_{1,2} &= -1\% \\ f_{2,2} &= -2\% \\ f_{3,2} &= 0\% \end{aligned}$$

(b) The prices of zero coupon bonds for various terms are as follows:

$5\rceil = £85\%$ $\quad$ $10\rceil = £65\%$ $\quad$ $15\rceil = £40\%$

Calculate Y_{15} and $F_{10,5}$

[10]

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12. (a) Calculate the 4 year par yield if the annual term structure of interest rates is

$$y_1 = 5\%, \quad y_2 = 5\tfrac{1}{4}\%, \quad y_3 = 5\tfrac{1}{2}\%, \quad y_4 = 5\tfrac{3}{4}\%.$$

(b) Calculate the gross redemption yield on a 5 year fixed interest security redeemable at par if the annual coupon is 2%, given

$$y_1 = 6\%, \quad y_2 = 6\%, \quad y_3 = 7\%, \quad y_4 = 8\%, \quad y_5 = 9\%.$$

[10]

13. (a) What are Redington's conditions for immunisation?

(b) Calculate the effective duration (volatility) for a bond redeemable at par in 5 years time with annual coupons of 4% at an interest rate of 3%.

(c) Calculate the convexity of the following assets using an interest rate of 10%.

(i) 5 year zero coupon bond

(ii) A level perpetuity payable annually in advance.

[10]