

Ollscoil na hÉireann, Gaillimh
National University of Ireland, Galway

GX 1506

Semester II Examinations, 2003/2004

Exam Code(s)	<u>2IF1, 3EL2</u>
Exam(s)	<u>IF1 B.Sc. (Information Technology)</u> <u>EL2 B.Sc. Degree (Applied Physics and Electronics) (Hons.)</u>
Module Code(s)	<u>CT214</u>
Module(s)	<u>Logical Foundations Of Computing</u>
Paper No.	<u>1</u>
Repeat Paper	<u>Special Paper</u>
External Examiner(s)	<u>Dr. Dave Johnson, Professor P. Nixon</u>
Internal Examiner(s)	<u>Professor G. Lyons, Professor T. Hurley</u> <u>Dr. E. Sköldberg</u> <u>Dr. M. Mc Gettrick</u>

Instructions

Answer 4 questions, two from each section.
All questions will be marked equally.
Please use separate answer books for sections A and B

Duration	<u>3hrs</u>
No. of Answer Books	<u>2</u>

Requirements

Handout	<u> </u>
MCQ	<u> </u>
Statistical Tables	<u> </u>
Graph Paper	<u> </u>
Log Graph Paper	<u> </u>
Other Material	<u> </u>

No. of Pages	<u>3</u>
Department(s)	<u> </u>

Please use separate answer books for Sections A and B.

SECTION A
Answer Only 2 Questions

- A1.** (i) Define what it means for two propositions to be *logically equivalent*.
(ii) Define what it means that a logical proposition is a *tautology* and a *contradiction*.
(iii) For each of the following propositions, determine if it is a tautology, a contradiction or neither:
- (a) $\neg(p \rightarrow \neg p)$
 - (b) $(\neg p \vee q) \leftrightarrow (p \rightarrow q)$
 - (c) $(p \wedge q) \vee (\neg p \wedge \neg q)$
- A2.** (i) Write down the truth tables for $p \vee q$, $p \wedge q$ and $\neg p$.
(ii) Use the method of resolution to check if the following arguments are valid:
- (a) $\{p \vee q, p \rightarrow s, q \rightarrow s\} \models s$
 - (b) $\{q, \neg p \leftrightarrow \neg q\} \models p$
- A3.** (i) Find the disjunctive normal form of the following propositions
- (a) $\neg p \wedge (\neg q \rightarrow r)$
 - (b) $p \leftrightarrow (q \wedge r)$
 - (c) $\neg(p \rightarrow q)$
- (ii) Using natural deduction, show the following
- (a) $\vdash (\neg p \rightarrow p) \rightarrow p$
 - (b) $\vdash (p \rightarrow q) \rightarrow (\neg q \rightarrow \neg p)$

SECTION B
Answer Only 2 Questions

- B1.** (i) Give an example of each of the following:
- (a) Idempotency of \vee
 - (b) Left Identity element of \rightarrow
 - (c) Absorption element of \wedge
 - (d) Distribution of \vee over \wedge
- (ii) Using the laws of Propositional Calculus, prove the following:
- (a) $p \rightarrow (q \rightarrow r) = q \rightarrow (p \rightarrow r)$
 - (b) $(p \vee q) \rightarrow (\neg p \wedge q) = \neg p$
 - (c) $((q \vee p) \vee r) \vee s = p \vee (q \vee (s \vee r))$
 - (d) $(p \rightarrow \neg q) \leftarrow r = (q \wedge r) \rightarrow \neg p$
- Give a reason for each step, and if you combine several steps into one, list all the laws used.
- (iii) Prove that *Hypothetical Syllogism* is a valid inference rule.

- B2.** (i) Construct two separate proofs using (a) the Deduction Theorem (b) Reductio ad Absurdum for each of the following (in each case give a reason for every step):

(a)

$$\{p \rightarrow q, r \rightarrow s, p \vee r\} \vdash \neg q \rightarrow s$$

(b) The argument:

If it rains on Monday it will be sunny and dry on Tuesday. If it is sunny or dry on Tuesday, then it will be cloudy on Wednesday. Therefore, if it rains on Monday it will be cloudy on Wednesday.

- (ii) Consider the following paradox:

S1: The Sun is in orbit around the Earth.

S2: Neither of these statements is true (i.e. neither S1 nor S2 is true).

Can a consistent set of truth values be assigned to S1 and S2? If so, state the assignment(s) and the resulting value(s) of S1.

- B3.** (i) For each of the expressions

(i) $\exists x : U_1 \bullet \forall y : U_2 \bullet P(x, y)$

(ii) $\forall x : U_1 \bullet \exists y : U_2 \bullet P(x, y)$

determine the resulting truth value when

(a) $U_1 = \phi \neq U_2$

(b) $U_2 = \phi \neq U_1$

(c) $U_1 = \phi = U_2$

(ϕ is the empty or null set $\{\}$)

- (ii) Let U be the set of all people ($x \in U$) and V be the set of all languages ($y \in V$). We define the following Atomic Predicates:

$S(x, y)$: person x speaks language y

$K(x)$: person x is Korean

$IE(y)$: y is an Indo-European language

Represent the following statements in Predicate Calculus:

(a) Korean people speak some Indo-European languages.

(b) All Indo-European languages are spoken by somebody.

(c) If everyone speaks every language, then no one speaks French.

(d) The only people who speak Korean are Korean.

(e) Non-Koreans do not speak all Indo-European languages.