

OLLSCOIL NA hÉIREANN, GAILLIMH
NATIONAL UNIVERSITY OF IRELAND, GALWAY

SEMESTER I EXAMINATIONS, 2003/2004

Module Code: **MA237**
Module: **PROBABILITY AND STATISTICS**
External Examiner Dr. T. C. Bailey
Internal Examiner Prof. J. P. Hinde

Instructions:

Duration: **Two Hours.**

Answer the 10 Questions in Section A (20 marks)
and

any 4 Questions from Section B (20 marks each).

Relevant tables and formulæ are supplied.

Question 1 is on the next page

Section A - Compulsory

Each part is worth 2 marks with a loss of 1 mark for a wrong answer.

1. Which of the following measures of spread is **least affected** by extreme values in a set of data ?
 - (a) range
 - (b) standard deviation
 - (c) inter-quartile range

2. The following data are scores on a 20-point test for a group of ten students.

18 15 12 6 8 2 3 5 20 10

What is the median score for these students?

- (a) 9
 - (b) 10
 - (c) 5
3. For any event A and its complement \bar{A} , which of the following is **not true**?
 - (a) $P(A \cup \bar{A}) = 0$
 - (b) $P(A) + P(\bar{A}) = 1$
 - (c) $P(A \cap \bar{A}) = 0$
4. If two events A and B are **independent** then which of the following is true?
 - (a) $P(A \cap B) = 0$
 - (b) $P(A \cup B) = P(A) + P(B)$
 - (c) $P(A \cap B) = P(A)P(B)$
5. The number of ways of selecting a captain and a vice-captain from a team of 11 players is
 - (a) 121
 - (b) 55
 - (c) 110
6. If a coin is tossed repeatedly n times, then the appropriate probability distribution for the number of heads is a
 - (a) Poisson distribution
 - (b) Binomial distribution
 - (c) Geometric distribution

Section A continued ...

7. Suppose that someone offers you a bet on the roll of a pair of fair dice. If you score exactly 7 you win €36. Otherwise you must pay €6. Your expected winnings would be
- (a) €6
 - (b) €1
 - (c) €0
8. For a random variable X with variance 16, the standard deviation of $2X + 5$ is
- (a) 32
 - (b) 13
 - (c) 8
9. If $Z \sim N(0, 1)$ and if $P(Z < k) = 0.95$, then k is
- (a) 1.960
 - (b) 1.645
 - (c) -1.960
10. The Binomial distribution $\text{Binomial}(16, 0.5)$ is well approximated by which of the following normal distributions?
- (a) $N(8, 4)$
 - (b) $N(8, 8)$
 - (c) $N(16, 4)$

Section B - Answer 4 Questions

1. An insurance company has collected the following data on the number of car thefts in a large city for a period of 25 days.

52	62	51	50	69
58	77	66	53	57
75	56	55	67	73
79	98	68	65	72
57	51	63	69	75

- (a) Construct a stem-and-leaf plot using intervals for the stems of width 5. (5)
 - (b) Find the five number summary. (5)
 - (c) Draw a boxplot. (5)
 - (d) Calculate the sample mean and sample standard deviation. (5)
2. (a) How many different ways are there of writing down (in sequence) the letters in the word
- MISSISSIPPI
- How many of these ways include the consecutive string of letters MISS? (2)
- (b) In drawing cards from a well-shuffled pack, what is the probability that
 - i) the first card drawn is an Ace? (1)
 - ii) the second card drawn is an Ace, given that the first card was an Ace? (2)
 - iii) the second card drawn is an Ace? (2)
 - (c) An accounts office has two clerks processing invoices. Clerk A does 60% of the work and has a 5% chance of making an error. Clerk B does the other 40% of the work and has a 10% chance of making an error. The supervisor checks a randomly chosen processed invoice,
 - i) what is the probability that it will be in error? (3)
 - ii) given that the invoice is in error, what is the probability that it was processed by B? (3)
 - (d) The distribution of blood types for a certain population is

35% type A 10% type B 50% type O 5% type AB

- If two people are chosen at random from the population, what is the probability that they have the same blood type? (4)

3. (a) On a particular stretch of road, 30% of cars exceed the 50km/hr speed limit. Fifteen cars pass a hidden police camera. Explain why a binomial distribution might be appropriate for the number of these cars that are speeding. (2)
- Find the probability that:
- i) exactly 3 of these cars are speeding; (2)
 - ii) at least one of the cars is speeding. (2)
- If a further fifteen cars pass the camera, what is the probability that there are exactly 3 speeding cars in this second group of fifteen cars **and** exactly 3 speeding cars in the first group of fifteen? (2)
- Is this the same as the probability of there being 6 speeding cars in the combined group of thirty cars? Explain. (2)
- (b) The number of seismic events in a certain region is Poisson distributed with a mean of 2 per year. Find the probability that
- i) there are no events in a given year; (2)
 - ii) there are at least two events in a given year; (2)
 - iii) there are no events over a five-year period; (3)
 - iv) over a five-year period there are no events in the first three years and exactly one event in the remaining two years. (3)
4. The joint probability mass function of two random variables X and Y is given by the following table:

		Y		
		-1	0	1
X	-1	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$
	0	$\frac{1}{6}$	$\frac{1}{6}$	$\frac{1}{6}$
	1	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$

- (a) Find the marginal probability functions for X and Y . (3)
- (b) What is the conditional probability that $X = 0$ given that $Y = 0$? (2)
- (c) Calculate the mean and variance of X and Y . (6)
- (d) Calculate the covariance between X and Y . (4)
- (e) Find the correlation between X and Y . (2)
- (f) Are X and Y independent? Give a reason. (3)

5. (a) The population of Statsville has heights that are normally distributed with a mean of 180cm and a standard deviation of 10cm.
- i) What is the probability that a randomly selected individual's height is more than 195cm? (2)
 - between 165 and 195cm? (2)
 - ii) What height is exceeded by all but the smallest 5% of the population? (2)
 - iii) In a room of 100 people, what is the probability that the average height of the people in the room is greater than 182cm? (4)
- (b) Calls arrive at an office switchboard from both internal (I) and external (E) lines. The numbers of calls from each of these sources are Poisson distributed with means of $\lambda_I = 40$ and $\lambda_E = 60$ per hour.
- Let N be the total number of calls arriving at the switchboard per hour. Assuming that the numbers of internal and external calls are independent, show that N has mean and variance 100. What is the distribution of N ? (3)
- Using the normal approximation to the Poisson distribution, calculate the probability that:
- i) there are 115 or more calls in a given hour; (3)
 - ii) there are between 90 and 110 calls, inclusive, in a given hour. (4)