

OLLSCOIL NA hÉIREANN, GAILLIMH  
NATIONAL UNIVERSITY OF IRELAND, GALWAY

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SEMESTER 1 EXAMINATIONS, 2003-2004

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FIRST YEAR FINANCIAL MATHEMATICS AND  
SECOND YEAR ARTS

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STATISTICS [MA112, MA227]

Dr. T.C. Bailey  
Professor T.C. Hurley  
Dr. J. McDermott

Time allowed: *Two* hours.  
Answer three questions.

1. (i) Here is a list of the number of accidents that occurred in the year 2000 at each of the 18 roundabouts in the Galway area:

8, 29, 31, 14, 35, 28, 12, 18, 24, 13, 6, 32, 2, 14, 26, 22, 32, 25.

Construct a stem-and-leaf display for the data, and find the median  $Q_2$  and the quartiles  $Q_1$  and  $Q_3$ .

- (ii) The scores for five students in a probability test are:

53, 44, 65, 58, 70.

Find the mean and the standard deviation of this *sample*.

- (iii) A national survey of car prices gives the mean price (in euros) of model A as 14,500 with standard deviation 400. The corresponding figures for model B are 14,200 and 500. The local dealers offer me model A at 13,500 and model B at 13,000. Which do you think is the better offer, and why?
- (iv) An executive of 4 is to be selected from a board consisting of 8 women and 5 men. In how many ways may this be done? What if the executive must have a majority of women?

p.t.o.

2. (i) Among the 60 students taking a certain course, 50 attend regularly, 35 are men, and 30 of the men attend regularly. What is the probability that a class representative chosen by lot will be a woman who does not attend regularly?
- (ii) State Kolmogorov's axioms for a probability model. Use the axioms to show that if  $A$  and  $B$  are events in a sample space such that  $A \subseteq B$  then

$$P(B \setminus A) = P(B) - P(A).$$

- (iii) The odds *against* event  $A$  are 4 to 1, and *against* event  $B$  they are 6 to 1. Find the odds *for*  $A \cup B$  if  $A$  and  $B$  are independent.
3. (i) Our team has a 70% chance of winning the big match if the day is dry, but only a 50% chance otherwise. The forecast puts the chance of a dry day at 60%. What is the probability that our team will win?
- (ii) Prove the law of total probability in the form

$$P(A) = P(B)P(A|B) + P(\overline{B})P(A|\overline{B}).$$

Hence prove the theorem of Bayes concerning  $P(B|A)$ .

- (iii) A test with a false-positive rate of 3% and a false-negative rate of 4% is used to screen for a serious disease that occurs in 2 per 1000 of the population. Given that John is found positive by the test, what is the probability that he has the disease? Given that Mary is found negative by the test, what is the probability that she does not have the disease?

p.t.o.

4. (i) In a certain restaurant, past experience suggests that 1 in 5 diners orders a vegetarian meal. What is the probability that in a party of 12 diners exactly 3 will choose the vegetarian option?
- (ii) The actual amount of coffee in a 100-gram jar has a normal distribution and a standard deviation of 0.5 grams. What should the mean fill of the jars be if at most 1% of the jars should contain less than 100 grams of coffee?
- (iii) The distribution of weights of all passengers on flights from Galway to Dublin has a mean of 160 lbs and a standard deviation of 15 lbs. Use the Central Limit Theorem to estimate the probability that the combined weight of 25 passengers on such a flight exceeds 4125 lbs.