Gx1973

OLLSCOIL na hÉIREANN, GAILLIMH

NATIONAL UNIVERSITY of IRELAND, GALWAY

WINTER EXAMINATIONS 2003

SECOND UNIVERSITY EXAMINATION

DISCRETE MATHEMATICS [MA 204]

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Time allowed: **Two** hours.
Full marks for **three** correct solutions.

1. (a) There are 100 pupils in a school. Forty pupils take French, forty pupils take German and forty pupils take Latin. Twenty pupils are taking **only** French, twenty are taking **only** Latin and fifteen are taking **only** German. In addition ten pupils are taking French and Latin.
(i) How many pupils are taking all three languages?
(ii) How many of the pupils are taking none of the three languages?

(b) Find the number of distinct permutations (arrangements) of all the letters in the word

COMBINATORIAL

Of these arrangements, how many
(i) have all the vowels together?
(ii) end in four consonants?

(c) General Incompetence and Major Disaster play a chess match. The first person to win three games wins the match. Assuming no drawn games, construct an appropriate tree diagram to find the number of ways in which the match may have occurred.

PTO

2. (a) Show that the number of non–negative integer solutions of

$$x_1 + x_2 + \cdots + x_k = n \quad \text{is} \quad \frac{(n+k-1)!}{n!(k-1)!}.$$

In how many ways can one distribute 10 one–euro coins to six students?
In how many ways can one distribute 10 one–euro coins to six students if no student receives more than 2 coins?

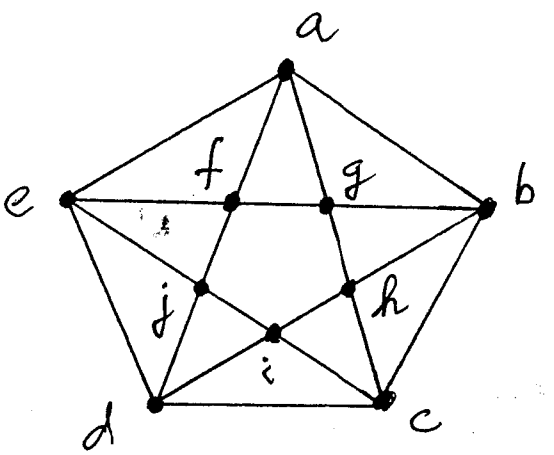
(b) A woman ascends a stairs either two steps at a time, or one step at a time. If W_n is the number of ways the woman can ascend n steps, show that

$$W_n = W_{n-1} + W_{n-2} \text{ for } n > 2.$$

Calculate W_9 using the recurrence above and that fact that $W_1 = 1$ and $W_2 = 2$.

(c) Use a binary search tree to sort the following list in alphabetical order:
matrix, eigenvalue, cofactor, symmetric, diagonal, inverse, elementary, determinant, adjoint, transpose, orthogonal, hermitian

3. (a) Define the term **eulerian graph**. Give a necessary and sufficient condition for a graph to be eulerian.
What is meant by the term **hamiltonian cycle**?
Determine whether or not the graph illustrated below is eulerian, (and if so, give an **eulerian trail**), also whether or not it is hamiltonian (and if so, give a hamiltonian cycle):



(b) Let T be a graph with n vertices and $n-1$ edges and no cycles. **Prove** that T is a tree.

continued

(c) Describe an algorithm for finding a spanning tree in a connected graph and use the algorithm to find such a spanning tree in the graph in part (a) above.

(d) Construct an ordered rooted tree for the algebraic expression

$$\sin^3(\pi t) - 2\cos(\pi/4)$$

and hence write the expression in Polish notation.

4. (a) For a connected planar map having V vertices, E edges and F faces, **prove** Euler's formula

$$V - E + F = 2.$$

Show that such a map has a vertex of degree at most 5.

(b) Show that if a graph G has no vertex of degree greater than k its chromatic number $\chi(G)$ is at most $k + 1$.

(c) The Dean of the Science Faculty wishes to schedule eight 1–hour lectures in (α) Mathematics, (β) Physics, (γ) Chemistry, (δ) Microbiology, (ϵ) Biochemistry, (ζ) Geology, (η) Zoology and (θ) Botany. Among the potential audience are people who wish to take the following pairs of subjects:

$$\alpha+\beta,\ \alpha+\gamma,\ \alpha+\delta,\ \alpha+\epsilon,\ \alpha+\theta,\ \beta+\delta,\ \beta+\eta,\ \beta+\theta,$$
$$\gamma+\zeta,\ \delta+\epsilon,\ \delta+\theta,\ \zeta+\eta,\ \epsilon+\theta,\ \eta+\epsilon,\ \delta+\zeta,\ \gamma+\theta$$

By applying the Welch–Powell algorithm for colouring the corresponding graph, or otherwise, how many hours are necessary in order that the 8 lectures can be given without clashes?