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OLLSCOIL NA hÉIREANN, GAILLIMH
THE NATIONAL UNIVERSITY OF IRELAND, GALWAY

SEMESTER I EXAMINATIONS 2003

B.Sc. BUSINESS INFORMATION SYSTEMS

MA208.I - QUANTITATIVE TECHNIQUES FOR BUSINESS 1

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Time allowed: Two hours
Answer four questions.

1. (a) Given the following input-output matrix for two industries X and Y:

	Input to X	Input to Y	Final Demand
Output from X	750	300	450
Output from Y	300	600	300

(i) Complete the above table using the assumption that total input = total output for each sector.

(ii) Write down the matrix of technical co-efficients.

(iii) Calculate the total output required from each industry if the final demands from X and Y change to 500 and 1000 units respectively.

(iv) Construct the new input-output table.

(b) A major telephone company (company X) has studied the tendency of users to switch from one carrier to another. The company has established that, over successive twelve-month periods, 20% of their customers will switch to a competing service while 30% of those who use the competing service will switch to X. Let x_n denote the proportion using company X and y_n the proportion using the competing service in year n (with x_0 and y_0 the initial proportions).

(i) Find the transition matrix, A, corresponding to the process such that

$$\begin{pmatrix} x_{n+1} \\ y_{n+1} \end{pmatrix} = A \begin{pmatrix} x_n \\ y_n \end{pmatrix} \text{ and show that } A^n \begin{pmatrix} x_0 \\ y_0 \end{pmatrix} = \begin{pmatrix} x_n \\ y_n \end{pmatrix}$$

(ii) Find the eigenvalues and eigenvectors of A and hence calculate A^n.

(iii) If company X presently controls 70% of the market, what percentage can it expect to control in the long term?

2. A company manufactures two types of couplings. Each coupling requires processing time on lathes, grinders and polishers. The number of machine-hours required for each type of coupling and the maximum hours available are given in the table below:

Model	Lathe	Grinder	Polisher
Type 1	2	8	5
Type 2	5	5	2
Maximum Hours	250	310	160

(a) If x represents the number of couplings of type 1 and y represents the number of type 2, write down three inequalities to represent the above constraints.

(b) Plot these inequalities and shade in the region of feasible solutions.

(c) Determine using the corner-point method, the number of each type that should be produced to maximize profit, if the profit per unit is €9 for type 1 and €10 for type 2.

(d) Calculate, also, the number of machine-hours which are not used when profit is maximized.

3. (a) Let $A = \begin{pmatrix} -2 & 5 & 1 \\ 3 & -1 & 4 \\ 4 & 0 & 1 \end{pmatrix}$

(i) Find C, the matrix of cofactors of A.

(ii) Calculate A*, the adjoint of A.

(iii) Calculate A.A* and hence write down $|A|$, the determinant of A.

(iv) Use (ii) and (iii) to write down A^{-1}, the inverse of A.

(b) Consider the system of equations:

$$\begin{aligned} -2x + 5y + z &= -4 \\ 3x - y + 4z &= 2 \\ 4x \quad + z &= 7 \end{aligned}$$

Rewrite this system as: $A\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -4 \\ 2 \\ 7 \end{pmatrix}$

where A is the matrix in (a) and hence solve the system.

4. (a) Differentiate *three* of the following functions with respect to x:

(i) $f(x) = e^{-2x} \sin x^2$ (ii) $f(x) = (3 - \sqrt{x})^6$

(iii) $f(x) = \dfrac{x^2 \ln x}{x^3 - 1}$ (iv) $f(x) = \sin^{-1}(\frac{1}{x})$

(b) The weekly demand function for a particular product is given by:

$$q = f(p) = 100 - p$$

where q is the quantity demanded and the price p is given in euro.

(i) Determine the total revenue function, R(p), where R is a function of p.

(ii) What is the total revenue at a price of €30 and how many units will be demanded at this price?

(iii) At what price will total revenue be maximized and what is the total revenue at this price?

5. (a) The total cost of producing q units of a certain product is described by the function

$$C(q) = 1{,}000 + 200q + 0.1q^2$$

where C is the total cost in euro.

(i) How many units should be produced in order to minimize the *average cost per unit*?

(ii) What is the minimum *average cost per unit*?

(iii) What is the *total cost* of production at this level of output?

(b) The total cost and revenue functions for a certain product are:

$$C(q) = 300{,}000 + 200q + 0.006q^2$$
$$R(q) = 1{,}000q - 0.01q^2$$

Using the marginal approach, determine the profit-maximizing level of output. What is the maximum profit?

6. (a) The nominal rate of interest on an investment is 6 per cent per annum. What is the effective annual interest rate (*APR*) if interest is compounded

(i) quarterly (ii) monthly (iii) continuously?

(Answer correct to 2 decimal places)

(b) Calculate, correct to 2 decimal places, the compound interest rate required for €10,000 to double in six years if interest is compounded annually.

(c) A mortgage loan of €100,000 is available at an annual interest 11.25%. If the period of the loan is 30 years compute:

(i) the monthly repayments if interest is compounded monthly

(ii) the total repayment and

(iii) the total interest paid on the mortgage.

Calculate the monthly payments if the mortgage is repaid in 20 years and compute the difference in interest paid.