

OLLSCOIL NA hÉIREANN, GAILLIMH
THE NATIONAL UNIVERSITY OF IRELAND, GALWAY

SEMESTER I EXAMINATIONS, 2003/2004

SECOND UNIVERSITY EXAMINATION

MATHEMATICS [MA211] CALCULUS 1

PASS

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Time allowed: *Two* hours.
Full marks for *three* questions.

1.

(a) Evaluate *three* of the following integrals:

$$(i) \int \frac{x^3 + 1}{x^3 - x} dx \quad (ii) \int \frac{x^2}{\sqrt{1 - x^2}} dx \quad (iii) \int \cos^5 x dx \quad (iv) \int \frac{2 dx}{x^2 + 2x + 2}$$

(b) Define the hyperbolic functions $y = \sinh x$ and $y = \cosh x$. Prove the identities

$$(i) \cosh(x + y) = \cosh x \cosh y + \sinh x \sinh y$$

$$(ii) \sinh 2x = 2 \sinh x \cosh x.$$

Prove that

$$\frac{d}{dx}(\cosh^{-1} \frac{x}{a}) = \frac{1}{\sqrt{x^2 - a^2}}.$$

p.t.o.

2. What does the *comparison test* for improper integrals state?

(a) Test for convergence *three* of the following improper integrals:

$$(i) \int_1^{\infty} \left(\frac{1}{x} + \frac{1}{x^2} \right) dx \quad (ii) \int_1^{\infty} \frac{1}{xe^x} dx \quad (iii) \int_{-1}^1 \frac{1}{x^3} dx \quad (iv) \int_1^{\infty} \frac{x+1}{x^3+x^4} dx$$

(b) Verify that each of the following improper integrals converges to the given value:

$$(i) \int_0^2 \frac{dx}{(x-1)^{1/3}} = 0 \quad (ii) \int_0^1 \sqrt{\frac{1+x}{1-x}} dx = \frac{\pi}{2} + 1$$

3. Solve the equations:

$$(a) \frac{dy}{dx} = \frac{x^2 - 2y}{x}, \quad \text{where } x \neq 0.$$

$$(b) \frac{dy}{dx} - \frac{xy^3}{\sqrt{1+x^2}} = 0, \quad \text{subject to } y(0) = 1.$$

$$(c) (2x - y + 1) \frac{dy}{dx} = 2x - y.$$

4. Find the general solution of *two* of the following second order equations:

$$(a) \frac{d^2y}{dx^2} + y = xe^{2x}$$

$$(b) \frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = e^x \sin x$$

$$(c) \frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = x^2 e^x$$