

OLLSCOIL NA hÉIREANN, GAILLIMH
NATIONAL UNIVERSITY OF IRELAND, GALWAY

SEMESTER 1 EXAMINATIONS 2003/2004

MATHEMATICS [MA284] — DISCRETE MATHEMATICS

Dr D. Johnson

Professor T. Hurley

Dr. G. Pfeiffer

Time allowed: *Two* hours.

Answer *three* questions.

1.

- (a) A committee of six people is to be chosen from eight married couples. (i) In how many different ways can the committee be chosen? (ii) How many committees contain exactly four women? (iii) How many committees contain at least four women? (iv) How many committees contain no couple? (v) How many committees contain exactly two couples?

- (b) Find the number of different arrangements of the ten letters in the word

IDENTITIES.

How many of these arrangements (i) start with the five vowels; (ii) end with the 2 Ts; (iii) start with the five vowels and end with the 3 Ts? In how many arrangements do the letters **I** appear together as **III**?

- (c) State the Binomial Theorem and use it to prove that

$$\sum_{k=0}^n (-1)^k \binom{n}{k} = 0,$$

unless $n = 0$.

PTO.

2

2.

- (a) Ten soccer teams play in a cup, and every two of them have to meet in exactly one game. Show that, at any given time, there are at least two teams which have played the same number of games in this cup.
- (b) What is a derangement of a set X ? Denote the number of derangements of an n -element set by $D(n)$. Use the formula $D(n) = n! \sum_{k=0}^n (-1)^k / k!$ to show that $D(n) = n \cdot D(n-1) + (-1)^n$ for $n \geq 1$. Use the latter formula to calculate $D(12)$.
- (c) How many numbers in $\{1, 2, \dots, 400\}$ are not divisible by 8, 9, 10 or 12?

3.

- (a) What is a tree? Show that if a graph $G = (V, E)$ is a tree then $|E| = |V| - 1$.
- (b) Let G be the graph whose vertices are the words in the phrase

the summer that men first walked on the moon

and where two words are joined by an edge if they have no letter in common. Draw a picture of G . (Hint: write out the phrase and draw the edges to on above the phrase, the edges to moon below the phrase.) Then, to every edge in G assign as its weight the sum of the lengths of the two words it joins. Describe briefly, Kruskal's Algorithm. Use Kruskal's Algorithm to find a minimum weight spanning tree in G . What is the weight of such a spanning tree?

- (c) Construct an ordered rooted tree for the algebraic expression

$$((A + B)(A - B) - 2A)/(A^2 + B^2)$$

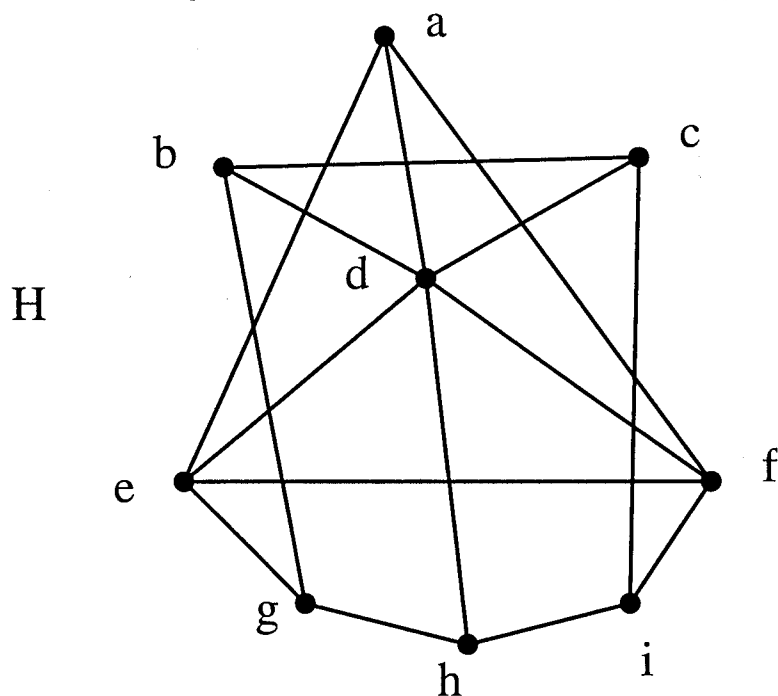
and write this expression in reverse Polish notation. Also, construct an ordered rooted tree for the prefix expression

$$* + + 10 2 / 9 3 - / 8 4 - 7 5$$

and write it as infix expression and determine its value.

4.

(a) Use the Welsh-Powell algorithm to determine a colouring of the graph H below.



What is the chromatic number $\chi(H)$?

- (b) Define what it means for a graph to be (i) bipartite, (ii) Eulerian, (iii) Hamiltonian. Is the graph H above (i) bipartite? (ii) Eulerian? (iii) Hamiltonian? Justify your answers.
- (c) (i) What is a planar graph? (ii) State Euler's formula for a connected planar graph. (iii) A planar graph is called a **Platonic graph**, if all vertices have the same degree $a \geq 3$ and all regions (or faces) have the same degree $b \geq 3$. Use Euler's formula to show that there are only 5 Platonic graphs.