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OLLSCOIL NA hÉIREANN, GAILLIMH

NATIONAL UNIVERSITY OF IRELAND, GALWAY

SEMESTER I EXAMINATIONS 2003/2004

SECOND UNIVERSITY EXAMINATION

MATHEMATICS

MA286 – ANALYSIS

HONOURS

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Time allowed: **Two** hours.

Attempt **three** questions.

The numbers in square brackets indicate the percentage of the total marks for the question that are associated to that particular part of the question.

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1. (a) **[30%]** Let

$$f(x,y) = \begin{cases} \dfrac{xy}{x^2+y^2} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0) \end{cases}$$

Show that f is not continuous at $(0,0)$.

(b) **[30%]** Let $f(x,y) = x^2y - xy + y^3$ and let $\mathbf{v} = (1,2)$. Compute $\frac{\partial f}{\partial \mathbf{v}}(2,1)$.

(c) **[40%]** Find and classify the critical point(s) of the function

$$f(x,y) = x^2 + 2y^2 + 3xy - 5x - 7y.$$

2. (a) [40%] Find the tangent line to the curve $x^3 + xy^2 + y^3 = 1$ at the point $(1, 1)$.

   (b) [60%] Let U be an open subset of $\mathbb{R}^2$ and let $f : U \to \mathbb{R}$. Suppose that f has continuous partial derivatives. Let $(a, b) \in U$. Show that for any $\mathbf{v} \in \mathbb{R}^2$,

$$\frac{\partial f}{\partial \mathbf{v}}(a, b) = \nabla f(a, b).\mathbf{v}$$

3. (a) [30%] Briefly describe the method of Lagrange multipliers for constrained optimisation.

   (b) [70%] A flat circular plate occupies the region $\{(x, y) \in \mathbb{R}^2 : x^2 + y^2 \leq 1\}$. The plate is heated so that the temperature at any point (x, y) is

$$T(x, y) = x^2 + xy + y^2 - x.$$

   Find the hottest and coldest points on the plate.

4. (a) [20%] What does it mean to say that a series is *absolutely convergent*?

   (b) [40%] Which of the following series is convergent?

      i. $\displaystyle\sum_{n=1}^{\infty} \frac{n^2 + 1}{n^3 + n + 1}$.

      ii. $\displaystyle\sum_{n=1}^{\infty} \frac{(3n + 1)^n}{n^n}$.

   (c) [40%] Give an example of a series, $\sum a_n$, with the property that $\sum a_n$ is convergent, but that $\sum a_n^2$ is divergent. You should prove that your example has the required properties.

5. (a) **[30%]** Find the radius of convergence of the following power series.

$$\sum_{n=1}^{\infty} \frac{(2x)^n}{2n+1}$$

(b) **[20%]** State Taylor's theorem. You are <u>not</u> required to give a proof.

(c) **[50%]** Let

$$f(x) = \sin 2x.$$

Using Taylor's theorem, or otherwise, show that f is a real analytic function.