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OLLSCOIL NA hÉIREANN, GAILLIMH
THE NATIONAL UNIVERSITY OF IRELAND, GALWAY

SEMESTER I EXAMINATIONS 2003

THIRD UNIVERSITY EXAMINATION IN ARTS AND SCIENCE

MATHEMATICS [MA301] ADVANCED CALCULUS

PASS

Dr. Dave Johnson
Professor T. C Hurley
Dr. M. Hayes

Time allowed: *Two* hours.

For those in the *Mathematical Studies* group, full marks for **four** questions.
For all other groups, full marks for **three** questions.
All questions carry equal marks.

1. (a) Which of the following series converge? Determine the limit of those series that converge:

$$\text{(i)} \sum_{n=3}^{\infty} \frac{2}{2^{2n}}, \qquad \text{(ii)} \sum_{n=0}^{\infty} \frac{\pi^n}{e^{2n}}, \qquad \text{(iii)} \sum_{n=5}^{\infty} \frac{1}{5}, \qquad \text{(iv)} \sum_{n=1}^{\infty} \frac{1}{(n+1)(n+3)}.$$

(b) Which of the following series converge:

$$\text{(i)} \sum_{n=1}^{\infty} \frac{\sqrt{n}}{n^3+3}, \qquad \text{(ii)} \sum_{n=1}^{\infty} \frac{n^2\sqrt{n}}{n^3+1},$$

explain your answer fully.

p.t.o.

2. (a) Find the centre, radius and interval of convergence of the following power series

$$\sum_{n=1}^{\infty} \frac{(2x-2)^n}{(n+5)(n+6)}$$

(b) Find the Taylor polynomial of degree 4, centred around $x = 3\pi$ for the function $f(x) = \operatorname{Sin} x$ and hence determine Sin(9.5) to four decimal places.

3. Let

$$J = \iint_D (\sqrt{x} + \sqrt{y}) dA$$

where D is the finite region between $y = x$ and $y = \sqrt{x}$

Evaluate J:

(a) integrating first with respect to x and then with respect to y;

(b) integrating first with respect to y and then with respect to x,

and verify that the answer in both cases is the same.

4. (a) Find the volume of the solid which is bounded above by $z = 3xy$, below by the trapezoid with vertices $(1,0), (3,0), (3,5)$ and $(1,1)$ in the x/y plane, and laterally by vertical planes.

(b) Evaluate the following double integral using polar coordinates

$$\iint_{x^2+y^2 \leq 9} x \, dA$$

p.t.o.

5. (a) Evaluate the line integral

$$\int_C (-y^3x + \frac{1}{3}x^3)dx + (\frac{1}{2}xy + x^2)dy$$

where C is the line segment from $(0,0)$ to $(0,5)$ followed by the line segment $(0,5)$ to $(5,5)$.

(b) Use Green's theorem to evaluate

$$\oint_C (e^{x^2} + \mathrm{Sin}(x^2) - y^2)dx + (x + \mathrm{Sin}(\mathrm{Cos}\, y^2))dy$$

where C is the boundary of the quarter disc

$$x^2 + y^2 \leq a^2, x \geq 0, y \geq 0$$

oriented counter clockwise.