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OLLSCOIL NA hÉIREANN, GAILLIMH
THE NATIONAL UNIVERSITY OF IRELAND, GALWAY

SEMESTER I EXAMINATIONS 2003

THIRD UNIVERSITY EXAMINATION IN FINANCIAL MATHEMATICS & ECONOMICS

ACTUARIAL MATHEMATICS [MA311] - LIFE CONTINGENCIES

HONOURS

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Time allowed: *Two* hours.
Answer **eight** questions.

1. Let $\ddot{a}_{x:\overline{n}|}$ denote the net single premium at age x of a temporary annuity due of €1 per year for n annual payments. Show

$$\ddot{a}_{x:\overline{n}|} = \frac{N_x - N_{x+n}}{D_x}$$

2. Let ${}_yP_{x:\overline{n}|}$ denote the net annual premium for a n-year endowment insurance policy of €1 (if the policy specifies that the net premium is payable in y equal annual amounts) issued at age x. Show

$${}_yP_{x:\overline{n}|} = \frac{M_x - M_{x+n} + D_{x+n}}{N_x - N_{x+y}}$$

p.t.o.

3. A policy issued to someone aged 30 promises to pay

(i) €10,000 if insurer dies before reaching 55.

(ii) €50,000 if insurer dies after reaching 55 but before 65, or if the insurer is alive at age 65.

Find the net annual premium if the policy is a 10-payment policy and if we assume 1958 CSO mortality, interest at $2\frac{1}{2}\%$ compounded annually and the death benefit is paid at the end of the year of death.

4. Show using the prospective method that the t^{th} terminal reserve for a n-year endowment policy (where the net premium is payable in n equal annual amounts) of €1 issued to a person aged x is

$$A_{x+t:\overline{n-t}|} - {}_nP_{x:\overline{n}|}\ddot{a}_{x+t:\overline{n-t}|}$$

5. Let T_x be the random future lifetime after age x of a life that survives to age x. Show its probability density function in ${}_tp_x\,\mu_{x+t}$.

6. Show

$$E[T_x^2] = 2\int_0^{w-x} t\,{}_tp_x\,dt$$

7. Show

$$\overline{a}_{x:\overline{n}|} = \int_0^n v^t\,{}_tp_x\,dt$$

p.t.o.

8. Suppose

$$e_{50} = 15.5,\ e_{51} = 15.4,\ e_{52} = 15.1 \text{ and } e_{53} = 15.$$

Calculate the probability that a life aged 50 will survive to age 53.

9. Assume for each integer y and $0 \leqq t \leqq 1$ that the probability density function ${}_tp_y\mu_{y+t}$ is constant. Show

$$\overline{A}_x = \frac{i}{\ln(1+i)} \frac{M_x}{D_x}.$$

10. Show

$$\ddot{a}_x^{(m)} = \frac{N_x}{D_x} - \frac{(m-1)}{2m}$$

if we assume for fixed $k \epsilon \{0, 1, ...\}$ that $v^{k+\frac{j}{m}}\ {}_{k+\frac{j}{m}}p_x$ is linear in j for $j = 0, 1, ..., m-1$.