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OLLSCOIL NA hÉIREANN, GAILLIMH
NATIONAL UNIVERSITY OF IRELAND, GALWAY

SEMESTER 1 EXAMINATIONS, 2003-2004

THIRD UNIVERSITY EXAMINATION IN ARTS & SCIENCE
H.DIP. APPLIED SCIENCE

MATHEMATICS [MA343] - GROUPS 1

HONOURS

Dr. Dave Johnson
Prof. T. Hurley
Dr. J. McDermott

Time allowed: *Two* hours.
Answer three questions.

1. (i) Let $C_n = \langle a \rangle$ be a cyclic group of order n. Show that if H is a subgroup of C_n then $H = \langle a^d \rangle$ for some divisor d of n.

(ii) Determine the elements in the subgroup $H = \langle a^{28} \rangle$ of the cyclic group C_{36}, and express H as $\langle a^d \rangle$ for a suitable divisor d of 36.

(iii) Show that $C_2 \times C_5$ is cyclic but $C_4 \times C_6$ is not.

2. Let G be a finite multiplicative group.

(i) Prove Lagrange's theorem: if H is a subgroup of G then its order $|H|$ divides $|G|$.
Deduce that if x is an element of G then its order divides $|G|$.

(ii) Now suppose that $|G| = 6$ and G is not cyclic. Show that G has an element, say a, of order 3 and an element, say b, of order 2. Deduce that $G = \{1, a, a^2, b, ab, a^2b\}$ and show that $ba = a^2b$. Illustrate how this determines all products in G by calculating ba^2 and $(ab)^2$. [You need not complete the table for G].

p.t.o.

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