

OLLSCOIL NA hÉIREANN, GAILLIMH
NATIONAL UNIVERSITY OF IRELAND, GALWAY

SEMESTER 1 EXAMINATIONS, 2003-2004

FOURTH UNIVERSITY EXAMINATION

MATHEMATICS [MA495] - LIFE CONTINGENCIES 2

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Dr. M. Hayes

Time allowed: *Two* hours.
Answer **THREE** questions.

In addition to this paper you should have available actuarial tables,
logarithmic tables and an electronic calculator which is
non-programmable and is not capable of storing text.

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1. (a) In the context of a mortality table being used as a stationary population model explain the meaning of l_x and L_x and give the relationship between the symbols.
- (b) An isolated island whose population is stationary experience English Life No. 12 tables - Males mortality. The total number of deaths in any year is 1013.
Find
 - (i) the total population
 - (ii) the number of people that reach 40 years of age in any year
 - (iii) the number of people that are aged 70 last birthday at any time
 - (iv) the number of people over age 75 last birthday now.
- (c) A company with a stationary staff employs 10,000 qualified tradesmen who serve a 5-year apprenticeship commencing on their 16th birthday. Find the number of apprentices who must be accepted for training each year if one-tenth fail the trade test at the end of the apprenticeship and tradesmen retire at age 65. Mortality is English Life Table No. 12 - Males.
- (d) A 58 year old man has just retired early and has received a €60,000 tax free lump sum. He is considering investing in the following options.
 - (i) Forty equal half-yearly installments payable to him or his estate (if decreased), the first payment payable in 10 years time assuming interest at 3.5% p.a. Find each payment.
 - (ii) An annuity-due payable monthly, guaranteed for 10 years assuming mortality on the A67/70 table select with a reduction of two years from his age and 4% interest p.a. There are expenses of 1% of each premium, €2 of each premium and €100 immediately. Find the monthly payment.
 - (iii) Two equal annual immediate annuities payable to himself and his wife (now aged 60) with expenses of half the man's first premium. The mortality basis is;
Males: A67/70 select
Females: a (55) female select
Interest at 4% p.a.
Find each annual payment.

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2. (a) Define the force of decrement by cause k at age x $(a\mu)_x^k$.
 (b) Show that the probability of not being removed by any decrement equals the product of the rates of not being removed for each decrement i.e.

$${}_n(ap)_x = \prod_{i=1}^m {}_np_x^i.$$

- (c) (i) A large company experienced the below listed mortality and retirement numbers from the number of staff in service at ages 60 and 61.

Age (x)	Number in service	Deaths	Retirement
	$(al)_x$	$(ad)_x^d$	$(ad)_x^r$
60	10,000	299	919
61	8,782	278	897
62	7,607		

Recent changes in working conditions are estimated to have reduced the annual independent rates of mortality by .005 at all ages. Assuming that the annual independent rates of retirement are unaltered, prepare a revised service table.

- (ii) Verify that your answer is correct using the formula in part (b) with $n = 2$.
 (d) Given the following combined table where s are spinsters and m are married women.

Age	Spinsters	Spinsters Dying	Spinsters Marrying	Married Women	Married Women Dying
x	$(sl)_x$	$(sd)_x$	$(sw)_x$	$(ml)_x$	$(md)_x$
20	7219	22	721	4319	13
21	6476	20	699	5027	17
22	5757	19	751	5709	22
23	4987			6438	

Find

- (i) The probability that a married woman age 20 will die within a year.
 (ii) The probability that a spinster age 20 will be alive and married 2 years later.
 (iii) Explain the formula used in (ii).

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3. Given the following single and joint annuity values which apply to a particular set of quadruplets who are now all alive and age 30.

$$a_{30} = 25.12 \quad a_{30:30} = 14.31$$

$$a_{30:30:30} = 9.72 \quad a_{30:30:30:30} = 5.41$$

- (a) Find the cost of providing one immediate annual annuity of €40,000 p.a. if all four quadruplets aged 30 remain alive, this reducing to €25,000 p.a. if one of the quadruples dies and reducing to €10,000 p.a. if there are only one or two quadruplets living.
- (b) (i) Explain the meaning of

$$A \frac{r}{x_1 x_2 x_3 \dots x_m}$$

- (ii) Explain why

$$A \frac{[r]}{x_1 x_2 \dots x_m}$$

does not exist.

- (iii) Find the single premium with interest at 3% for an assurance of €1,000 payable at the end of the year of death of the third life to fail out of the four quadruplets now aged 30.
- (c) Explain the meaning and use the Z method to express the following expressions in terms of certain period annuities and single and joint life annuities.

$$a_{\overline{xy}:\overline{zw}}$$

$$a_{\overline{xyzw}}$$

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4. (a) For both past and future service, derive the formulae, (defining all symbols used) for the value of an ill-health pension of $\frac{1}{k}$ times average salary per annum for each year of service.
- (b) Using the pension fund tables, find the value of the following pensions arising from past and future service to a person aged 50 who joined the pension scheme 20 years ago. Use an interest rate of 4% p.a.
- (i) €100 p.a. illhealth pension for each year of service
 - (ii) €1,000 p.a. retirement pension for each year of service
 - (iii) An illhealth pension of $\frac{1}{60}$ th of average salary for each year of service given that he has earned €500,000 to date with this company and he will earn €50,000 over the next year.
- (c) Using the pension fund tables, find the value using an interest rate of 4% p.a. of the future pension contributions that a 25 year old employee will contribute on a current salary of €25,000 p.a. The employee contributions are 4% of his total salary less 500 p.a. i.e. .04 times (salary - 500) each year.