

OLLSCOIL NA hÉIREANN, GAILLIMH  
NATIONAL UNIVERSITY OF IRELAND, GALWAY

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EASTER EXAMINATIONS 2004

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THIRD UNIVERSITY EXAMINATION

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MATHEMATICAL METHODS [MM351/3]

Time allowed: *Three* hours.

Answer *three* questions from each section.

Use separate answer books for each section.

SECTION A

Dr. Dave Johnson,  
Professor T. C. Hurley,  
Dr. R. A. Ryan

1. (a) Define the terms *Eulerian* and *Hamiltonian* as applied to graphs. Show that, in an Eulerian graph, every vertex has even degree.  
(b) Describe the *Bridges of Königsberg Problem* and show that it has no solution.  
(c) Describe the *Knight's Tour Problem* and show that there is no solution on a  $n \times n$  board if  $n$  is odd.

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2. (a) Explain the terms *tree*, *spanning tree* and *minimum spanning tree*. The following table shows the distances in miles between Athlone (A), Dublin (D), Galway (G), Limerick (L), Sligo (S) and Wexford (W). Use *Prim's Algorithm* to find a fibre optic network connecting these locations that uses the least amount of cable.

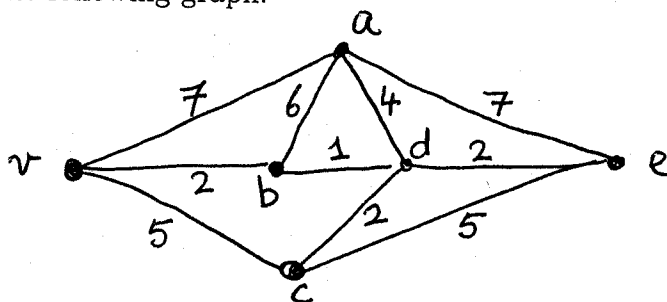
	A	D	G	L	S	W
A		76	56	73	71	114
D	76		132	121	135	96
G	56	132		64	85	154
L	73	121	64		144	116
S	71	135	85	144		185
W	114	96	154	116	185	

- (b) Using a binary decision tree, explain why any algorithm that sorts a list of  $n$  numbers must require  $O(n \log_2 n)$  operations. (You may use Stirling's formula:  $n! \sim \sqrt{2\pi n}(n/e)^n$ .)

Describe the *Heapsort* algorithm and use it to sort the following list into ascending order:

19, 2, 20, 1, 11, 10, 8, 12, 4.

3. (a) Apply the *Shortest Path Algorithm* to find the shortest path from vertex  $v$  to vertex  $e$  in the following graph:



- (b) State and prove the *Art Gallery Theorem*. Sketch a gallery for which four guards (and no fewer) are required to watch all the walls.
4. (a) Prove *Euler's Formula*,  $v - e + f = 2$ , for a connected planar graph. Kuratowski's Theorem says that every non-planar graph contains one or other of two particular non-planar graphs. Sketch these two graphs.
- (b) Define the *Chromatic Polynomial*,  $P_G(k)$ , of a graph  $G$ , and explain how it can be used to determine the *chromatic number* of  $G$ . Find the chromatic polynomial for the graph  $K_4$ .

### Section B

MM353/351

Professor B. Straughan;  
Dr. M. S. Ó Conphaola;  
Professor M. F. McCarthy.

1. Solve the equation

$$\frac{\partial u}{\partial t} = 4 \frac{\partial^2 u}{\partial x^2} : 0 \leq x \leq 10$$

subject to the boundary condition

$$\frac{\partial u(0, t)}{\partial x} = \frac{\partial u(10, t)}{\partial x} = 0$$

and the initial condition

$$u(x, 0) = \begin{cases} 60, & 0 \leq x < 5 \\ 0, & 5 \leq x \leq 10 \end{cases}$$

2. Let  $T(r, \theta)$  denote the steady state temperature in the region

$1 < r < c$ ,  $0 < \theta < \pi$ . The steady state temperature in the region is governed by the equation

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{1}{r^2} \frac{\partial^2 T}{\partial \theta^2} = 0, \quad 1 < r < c, \quad 0 < \theta < \pi.$$

If  $T = T_0$  when  $r = c$  and  $T = 0$  on all other boundaries, determine the temperature at each point  $(r, \theta)$  of the region.

3. Find the solution of the one dimensional wave equation

$$\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$$

in the region  $0 \leq x \leq L, t > 0$  which satisfies the boundary conditions

$$y(0, t) = y(L, t) = 0, \text{ for all } t > 0$$

and the initial conditions

$$y(x, 0) = f(x) \quad ; \quad \frac{\partial y}{\partial t}(x, 0) = g(x).$$

In the particular case when  $g(x) = 0$  show that the solution is equivalent to two progressive waves moving in opposite directions.

4. Solve the equation

$$\frac{\partial u}{\partial t} = 2 \frac{\partial^2 u}{\partial x^2}, \quad x > 0, \quad t > 0$$

$$u(x, t) \text{ bounded for } x > 0, \quad t > 0$$

subject to the initial condition

$$u(x, 0) = e^{-x}, \quad x > 0$$

and the boundary condition

$$u(0, t) = 0$$

using the separation of variables and the Fourier integral.

[A function  $f(x)$  may be written as

$$f(x) = \frac{1}{\pi} \int_0^\infty [A(\alpha) \cos(\alpha x) + B(\alpha) \sin(\alpha x) d\alpha]$$

where

$$A(\alpha) = \frac{1}{\pi} \int_{-\infty}^\infty f(\bar{x}) \cos \alpha \bar{x} d\bar{x},$$

$$B(\alpha) = \frac{1}{\pi} \int_{-\infty}^\infty f(\bar{x}) \sin(\alpha \bar{x}) d\bar{x} .]$$