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NATIONAL UNIVERSITY OF IRELAND, GALWAY

SUMMER EXAMINATIONS 2004

FIRST SCIENCE EXAMINATION

MA101 - CALCULUS

PASS

First Paper

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Time allowed: **Three** hours.
Answer *six* questions only.

1. (a) Evaluate the following limits:

(i) $\lim_{x\to 0} \dfrac{1-\operatorname{Sin} x}{e^x+1}$ (ii) $\lim_{x\to 1} \dfrac{x^8-3x^2+2}{x^{10}+2x^5-4x+1}$ (iii) $\lim_{x\to\infty} \dfrac{6x^3+2x+1}{5x^4+5x+3}$

(b) Differentiate the following with respect to x:

(i) $\dfrac{\operatorname{Sin} x}{x^2+e^x}$ (ii) $(x^8+\operatorname{Sin} x)^{17}$ (iii) x^{x^2+1}.

p.t.o.

2. Let $h(x) = 3x^5 - 5x^4 + 2$.

(a) Find the intervals on which h is increasing, and the intervals on which it is decreasing.

(b) Find the intervals on which the graph of h is concave up, and the intervals on which it is concave down.

(c) Find all relative extrema of h and points of inflection of the graph of h.

(d) Sketch the graph of h, labelling the y-axis intercept and the points found in (c). *Do not use graph paper or a table of values.*

3. (a) Explain (define) the statement "$f(x)$ is continuous at $x = a$".

(b) Suppose

$$f(x) = \begin{cases} kx^2 + 2x + 1, & x \geq 5 \\ 2x + 3, & x < 5. \end{cases}$$

For what value of k is f continuous at $x = 5$?

(c) Suppose

$$f(x) = \begin{cases} x^7 + 2x^3 + 4, & x \geq 1 \\ 3x^5 + 4, & x < 1 \end{cases}$$

(i) Is f continuous at $x = 1$?
(ii) Find $f'_+(1), f'_-(1)$.

p.t.o.

4. (a) State the Mean Value Theorem.
   (b) Prove that if $f'(x) < 0$ for $x \in (a, b)$ then f is decreasing on (a, b).
   (c) Let

$$f(x) = (1 + 4x)^{\frac{3}{2}} - (1 + 6x)$$

   (i) Show that f is increasing on $(0, \infty)$.
   (ii) Deduce from (i) that $(1 + 4x)^{\frac{3}{2}} > 1 + 6x$ for $x > 0$.

5. (a) Find the equation of the tangent line to the curve $f(x) = \frac{e^x}{x^2+1}$ at the point $(0, 1)$.
   (b) Sketch the region enclosed by the graphs of $y = x^2$ and $y = \sqrt{x}$.
   (i) Find the area of this region.
   (ii) Find the volume of the solid generated by revolving this solid about the x-axis.

6. Answer only *three* of the following:
   (a) Use the method of substitution to find

$$\int 2x \cos(x^2 + 1) dx$$

   (b) Use partial fractions to find

$$\int \frac{4}{(x-1)(x-3)} dx$$

   (c) Use integration by parts to find

$$\int_0^{\frac{\pi}{2}} x \cos x dx$$

   (d) Use the Fundamental Theorem of Calculus to find

$$\frac{d}{dx} \int_0^x \frac{\cos t}{\sqrt{t^2 - 1}} dt$$

7. (a) Use logarithmic differentiation to find $f'(x)$ if

$$f(x) = \frac{(x^2+3)^2(x^3-2x+1)^5}{\sqrt{x^3-2x}}$$

(b) Use partial fractions to find

$$\int \frac{2-3x}{(x-3)(x^2+1)}dx$$

8. (a) Solve the differential equation (by integrating factor method):

$$\frac{dy}{dx} + 3xy = (x-2)e^{-\frac{3}{2}x^2}$$

subject to $y = 2$ when $x = 0$.

(b) The population y of a certain town at any time t satisfies the differential equation

$$\frac{dy}{dt} = ky.$$

In 1995 ($t = 0$) the population of the town was 100 and in 2001 ($t = 6$) the population had grown to $1,000$.

(i) Find k.

(ii) In which year will the population reach $10,000$?