

OLLSCOIL NA hÉIREANN, GAILLIMH
NATIONAL UNIVERSITY OF IRELAND, GALWAY

SUMMER EXAMINATIONS, 2004

FIRST COMMERCE EXAMINATION

MATHEMATICS [MA130]

MA131 — CALCULUS

First Paper

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Time allowed: *Three hours.*

Answer six questions.

1. (a) Let $y = \frac{x-1}{2-3x}$. Find $\lim_{x \rightarrow \infty} y$ and $\lim_{x \rightarrow -\infty} y$. State the vertical and horizontal asymptotes (if any) of the graph of y .

(b) Evaluate the following limits:

- (i) $\lim_{x \rightarrow 2} \frac{x^3 - 5x + 2}{x^2 - 4}$
 (ii) $\lim_{x \rightarrow \infty} \frac{x^2 - 4}{x^3 - 5x + 2}$
 (iii) $\lim_{x \rightarrow 1} \frac{\ln(x^2 - x + 1)}{x^2 - x + 1}$
 (iv) $\lim_{x \rightarrow 1} \frac{e^{x^2}}{x - 1}$

2. (a) Differentiate $x^2 - 2x$ from first principles.

(b) Differentiate the following functions with respect to x :

- (i) $(3x^4 - 2x + 7)^{15}$
 (ii) $(x - 2)\sqrt{x^2 - 9}$
 (iii) $\frac{e^{x^2}}{\ln(x + 1)}$
 (iv) $\ln(e^{x^2 + 2x + 1})$

p.t.o.

3. (a) The monthly demand function for a particular product is

$$q = f(p) = 91,000 - 35p,$$

where q is stated in units and p is stated in euro. Determine the quadratic total revenue function $R = g(p)$. What does the total revenue equal at a price of €2000? How many units will be demanded at this price? At what price will total revenue be maximized? What is total revenue at this price?

- (b) Let

$$f(x) = \frac{x^4}{2} - \frac{8x^3}{3} + 4x^2.$$

Find $f'(x)$ and the values of x for which $f'(x) = 0$. Find the ranges of values of x for which $f(x)$ is increasing and the ranges of values of x for which $f(x)$ is decreasing. Find the critical points of f and also find the nature of these critical points. Sketch the graph of $y = f(x)$.

4. (a) The total cost of producing q units of a certain product is described by the function

$$C = 800 + 13q + 0.05q^3.$$

- (i) Determine how many units q should be produced in order to minimize the average cost per unit.
 - (ii) What is the minimum average cost per unit?
 - (iii) What is the total cost of production at this level of output?
- (b) The total cost and revenue functions for a certain product are

$$C(q) = 5,000,000 + 250q + 0.002q^2$$

$$R(q) = 1,250q - 0.005q^2$$

Find the marginal cost and marginal revenue. Using the marginal approach, determine the profit-maximizing level of output. What is the maximum profit?

5. (a) Evaluate the following indefinite integrals:

$$(i) \int \left(\frac{4}{x^3} + \sqrt{x} - 4x^4 \right) dx, \quad (ii) \int e^{2-3x} dx, \quad (iii) \int \frac{x^2 - 1}{x^3 - 3x + 7} dx.$$

- (b) Use integration by parts to evaluate the integral

$$\int x\sqrt{4x+3} dx.$$

- (c) Use the method of partial fractions to evaluate the integral

$$\int \frac{4x+7}{x^2+x-2} dx.$$

p.t.o.

6. (a) Find the general and particular solutions of the following differential equations:
- $\frac{dy}{dx} = 3x^4 + x^3, f(1) = 10.$
 - $\frac{d^2y}{dx^2} = 6x^2 + 2x + 3, f'(0) = 2, f(0) = -1.$
- (b) Find the area of the region contained between the graphs of the functions $f(x) = 4 - x^2$ and $g(x) = 2x + 4$.
7. (a) Evaluate the definite integral $\int_0^8 (x^2 + 1) dx$.
- (b) Find an approximate value for this integral using the Rectangle Rule and Simpson's Rule, subdividing into four intervals in each case. Which of these approximations is more accurate?
- (c) Answer **either** (i) **or** (ii):
- Describe the Trapezoidal Rule and derive the formula for this rule.
 - A solid of revolution is obtained by rotating around the x -axis the portion of the curve $y = 9 - x^2$ for x between 0 and 3. Find the volume of this solid.
8. (a) The population of a certain species appears to be growing at an exponential rate. Ten years ago, the population was 2,500. This year, the population is 5,000. Find the exponential growth function which describes the population P as a function of time t , and estimate how long it will take for this population to reach 10,000
- (b) Evaluate the following definite integrals:
- $\int_{-1}^0 xe^{3x^2} dx,$
 - $\int_0^2 (x + 1) dx - \int_2^0 (1 - 2x^2) dx.$