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NATIONAL UNIVERSITY OF IRELAND, GALWAY

SUMMER EXAMINATIONS, 2004

FIRST COMMERCE EXAMINATION

MATHEMATICS [MA130]

MA133 — *ALGEBRA*

Second Paper

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Time allowed: *Three* hours.
Answer six questions.

1. (a) The value of a machine is expected to decrease at a linear rate over time. Two data points indicate that the value of the machine at $t = 0$ (time of purchase) is € 60,000, and its value in 3 years is € 49,500. Determine the slope-intercept equation (of form $V = mt + k$) that relates the value V of the machine to its age t in years. Interpret the meaning of the slope. Find the t-intercept and interpret its meaning.

(b) The supply and demand functions for a product are $q_s = p^2 - 600$ and $q_d = 2p^2 - 80p + 1000$. Determine the market equilibrium price and quantity.

2. (a) Use Gaussian Elimination (only) to find the solution to

$$\begin{array}{rcrcrcr} x & + & y & + & z & = & 6 \\ 2x & - & y & + & 3z & = & 9 \\ 4x & + & 5y & - & 10z & = & -16 \end{array}$$

(b) Compute the determinant of

$$D = \begin{pmatrix} 3 & 3 & 6 & 9 \\ 6 & -3 & -2 & 0 \\ 2 & 7 & -2 & 0 \\ 1 & -2 & -3 & 0 \end{pmatrix}.$$

p.t.o.

3. (a) Find the inverse of the following matrix using the cofactor method

$$A = \begin{pmatrix} 1 & 1 & 0 \\ 2 & 0 & -1 \\ -3 & 6 & 4 \end{pmatrix}.$$

(b) Consider the system of equations

$$\begin{array}{rcrcrcl} x & + & y & & & = & 1 \\ 2x & & & - & z & = & 1 \\ -3x & + & 6y & + & 4z & = & 1 \end{array}$$

Rewrite this as

$$A \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

where A is the matrix as in part (a). By multiplying through (on the left) by the inverse of A, solve the system of equations.

4. (a) Use the Corner-Point Method to find the maximum and minimum values of the function $z = 3x_1 - 2x_2$ in the region defined by the following constraints:

$$\begin{aligned} x_1 + x_2 &\leq 12 \\ 3x_1 + x_2 &\leq 18 \\ x_1 &\geq 1 \end{aligned}$$

(b) Find the inverse of

$$A = \begin{pmatrix} 1 & 4 & 5 \\ 1 & 3 & 1 \\ 1 & 0 & 2 \end{pmatrix}$$

using *row operations only.* (DO NOT use the cofactor method.)

5. (a) Suppose an investment account earns interest of 6% per year, compounded every month. If € 10000 is put into the account in 2003, when will the investment be worth € 35000?

(b) What is the effective annual interest rate of an investment at a nominal rate of 16% per year, if interest is compounded continuously?

(c) If € 2000 is to grow to € 5000 over a 12-year period, at what annual rate of interest must it be invested, if compounding is quarterly?

p.t.o.

6. (a) A person wants to deposit €10,000 per year for 6 years in a savings account. If interest is earned at the rate of 4% per year, find the amount to which the deposits will grow by the end of the 6 years if:

   (i) Deposits of €10,000 are made at the end of each year with interest compounded annually;

   (ii) deposits of €5,000 are made at the end of each six month period with interest compounded semiannually.

   (b) Find the monthly payment necessary to repay a €20,000 car loan over 5 years if interest is 18% per year, compounded monthly. How much interest will be paid over the 5-year period?

7. **Formulate** the following Linear Programming Model. You are **not** required to find the solution.

   A coffee manufacturer mixes four different types of coffee beans into three blends of coffee. The four component beans cost €0.65, €0.80, €0.90 and €0.75 per pound, respectively. The weekly availability of the four components are 80,000, 40,000, 30,000 and 50,000 pounds, respectively. The manufacturer sells the three blends for €1.25, €1.40 and €1.80 per pound, respectively. Weekly output should include at least 50,000 pounds of blend 3.
   The following blending restrictions must be adhered to:

   (i) Component 2 should constitute at least 30% of blend 3 and at most 20% of blend 1.

   (ii) Component 3 should constitute exactly 25% of blend 3.

   (iii) Component 4 should constitute at least 40% of blend 1 and at most 18% of blend 2.

   The objective is to determine the number of pounds of each component that should be used in each blend so as to maximize weekly profit.

8. The following table contains the data for a Transportation Problem. The table lists the cost of shipping one unit from each of three origins to each of three destinations. Also listed are the supply capacities of each of the origins and the demands at each destination.

	Destination			
Origin	1	2	3	Supply
1	30	40	25	300
2	20	30	10	100
3	35	15	20	100
Demand	150	125	225	

   (i) Use the Northwest Corner Method to find an initial solution and find the cost of this solution.

   (ii) Use the Stepping Stone Algorithm to find an optimal solution. What is the cost of your solution?