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OLLSCOIL NA hÉIREANN, GAILLIMH

NATIONAL UNIVERSITY OF IRELAND, GALWAY

SUMMER EXAMINATIONS 2004

FIRST ENGINEERING AND INFORMATION TECHNOLOGY EXAMINATION

MATHEMATICS

MA151 – CALCULUS

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Time allowed: **Three hours**

Answer **six questions**

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1. (a) Evaluate the following limits:

   i. $\lim\limits_{x\to 2} \dfrac{x^3-8}{2x^2-2x-4}$.

   ii. $\lim\limits_{x\to\infty} x(\sqrt{x^2+1}-\sqrt{x^2-1})$.

   iii. $\lim\limits_{t\to\infty} \dfrac{3t^2-5t+4}{t-2t^2}$.

   (b) Sketch the graph of the following function. Do not use graph paper.

$$f(x) = \frac{x^2+4x+3}{x^2+x-2}.$$

2. (a) Find the values of x for which

$$|x-1| + |x-2| > 4$$

(b) For what values of k is the following function continuous?

$$f(x) = \begin{cases} x^2 - kx & \text{if } x < 1 \\ 2kx + 1 & \text{if } x \geq 1 \end{cases}$$

(c) Using the definition of a limit, show that

$$\lim_{x \to 1} \frac{x^2 - 1}{x - 1} = 2$$

3. (a) Compute the derivatives with respect to x of two of the following functions:

i.

$$f(x) = (\sqrt{x} + \frac{1}{\sqrt{x}}) \sin x$$

ii.

$$g(x) = \cos^3 (3 + x^3)$$

iii.

$$h(x) = \frac{\sin 2x}{\sin x + \cos x}$$

(b) Using logarithmic differentiation or otherwise to compute the derivative of

$$\frac{\sqrt[4]{x^2 - 1}(4x - 1)^3}{(2x - 1)^2 (4x)}$$

(c) Find $\frac{dy}{dx}$ when

$$\sin x + \cos y = \sin x \cos y$$

p.t.o.

4. (a) At noon, ship A is 100 km west of ship B. Ship A is sailing south at 35 km/h and ship B is sailing north at 25 km/h. How fast is the distance between the ships changing at 4pm.

(b) Find the dimensions of a closed top oil cylinder manufactured from $2m^2$ of metal if the volume is to be maximized.

5. (a) State the Fundamental Theorem of Calculus.

(b) Use this theorem to show that

$$\frac{d}{dx}\int_{x}^{\sin x}\frac{t}{\sqrt{1-t^2}}\,dt = \sin x - \frac{x}{\sqrt{1-x^2}}$$

(c) (i) Estimate $\int_0^1 x^2 dx$ by computing the upper and lower Riemann sums $U(x^2, P_2)$ and $L(x^2, P_2)$ respectively.

(ii) Now estimate the above integral by computing $U(x^2, P_4)$ and $L(x^2, P_4)$, and verify that

$$L(x^2, P_2) < L(x^2, P_4) < \frac{1}{3} < U(x^2, P_4) < U(x^2, P_2)$$

(Assume all subintervals are of equal width.)

6. (a) Evaluate *two* of the following integrals:

(i) $\displaystyle\int \frac{dx}{x\log x}$ (ii) $\displaystyle\int \frac{(1+t)}{t\sqrt{t^2-1}}\,dt$ (iii) $\displaystyle\int \sec^3 x\,dx$

(b) Establish the reduction formula:

$$\int x^m(\log x)^n\,dx = \frac{x^{m+1}(\log x)^n}{m+1} - \frac{n}{m+1}\int x^m(\log x)^{n-1}\,dx$$

and use it to evaluate $\int (x\log x)^2\,dx$.

p.t.o.

7. (a) Find the area enclosed by the curves

$$y = x^4 \text{ and } x = y^4$$

(b) Find the length of the curve

$$x = \sin^3 t,\ y = \cos^3 t,\ 0 \leq t \leq \frac{\pi}{2}.$$

(c) Find the volume generated when the curve $y = \sqrt{x}$, $1 \leq x \leq 4$ is rotated about the line $y = 1$.

8. Find the general solution of <u>three</u> of the following differential equations:

(a) $\frac{dy}{dx} = (1 + x)(1 + y^2)$

(b) $\frac{dy}{dx} = \frac{y-4x}{x-y}$

(c) $x\frac{dy}{dx} + 2y = 4x^2$

(d) $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 6y = x^2 + 1$