Gx 2010

OLLSCOIL NA hÉIREANN, GAILLIMH
NATIONAL UNIVERSITY OF IRELAND, GALWAY

SUMMER EXAMINATIONS, 2004

FINANCIAL MATHEMATICS AND ECONOMICS

CS204 - ALGORITHMS

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Time allowed: *Two* hours.
Full marks for *three* questions.

1. (a) What does it mean to say that an algorithm *correctly computes* a certain function?

Show that $z + xy$ is a loop invariant for the algorithm below and prove that the algorithm correctly computes the function $f(m,n) = mn$, where m,n are strictly positive integers.

$$\begin{array}{lll} x := m; & y := n; & z := 0; \\ \text{while} & y \neq 0 & \text{do} \\ & z := z + x & \\ & y := y - 1 & \\ p := z & & \end{array}$$

Discuss whether the condition 'strictly positive' is necessary for integers m, n in order for the algorithm to correctly compute their product.

(b) Illustrate *clearly* the sorting algorithm Tournament sort by using it to arrange the numbers 3, 7, 6, 9, 5, 2, 4 in natural order.

Prove that Tournament sort has (worst-case) complexity $0(n \log n)$. (Hint: Assume first that $n = 2^k$.)

2. (a) What is meant by an algorithm being (i) partially correct (ii) correct?

Show that $gcd(x, y)$ is a loop invariant for the algorithm below and prove that the algorithm correctly computes the function $f(m, n) = \gcd(m, n)$, where m,n are strictly positive integers.

$$
\begin{array}{lll}
x := m; & y := n; & \\
\text{while} & x \neq y & \text{do} \\
 & \text{if} & x > y \\
 & \text{then} & x := x - y \\
 & \text{else} & y := y - x \\
f := z & &
\end{array}
$$

If m and n are allowed to be non-negative integers, explain why the algorithm is partially correct, but *not* correct.

(b) Given a binary tree of height h with k leaves, it is true that $k \leq 2^h$. Use this fact to prove that every comparison-based sorting algorithm has complexity at least $0(n \log n)$.

3. (a) The algorithm below is supposed to compute the quotient q and remainder r when m is divided by n where m and n are positive integers. Find a loop invariant and prove that the algorithm is correct.

$$
\begin{array}{lll}
x := m; & y := n; & z := 0; \\
\text{while} & x \geq y & \text{do} \\
 & x := x - y; & \\
 & z := z + 1; & \\
q := z; & & \\
r := x & &
\end{array}
$$

(b) Illustrate the sorting algorithm Mergesort by using it to arrange the numbers 3, 7, 6, 9, 5, 2, 4 in natural order.

Prove that Mergesort has (worst-case) complexity $0(n \log n)$.

4. (a) Define the following terms with respect to a given algorithm:

   (i) complexity

   (ii) worst-case complexity

   (iii) average complexity

   What is meant by saying "$c(n)$ is $0(n \log n)$"?

   (b) Illustrate the sorting algorithm Tree sort by using it to sort the sequence of letters

   C O N F I G U R E

   in alphabetical order.

   Show that Tree sort has *average* complexity $0(n \log n)$.