Gx 2013

OLLSCOIL NA hÉIREANN, GAILLIMH

NATIONAL UNIVERSITY OF IRELAND, GALWAY

SUMMER EXAMINATIONS 2004

B.A., B.Sc. and Third Science Examination

Computer studies

CS304 = (MA325 + MA326) = CS310 = Second paper of CS320

Dr. Dave Johnson
Prof. T. Hurley
Dr. E. Sköldberg

Time allowed: *three* hours.
Candidates taking CS304 should answer four questions, two from each section.
Candidates taking only MA325 should answer two questions from section A.

Section A

Answer only two questions

1. (a) Define what it means for two propositions to be *logically equivalent.*

   (b) Define what it means for a logical proposition to be a *tautology* and a *contradiction* respectively.

   (c) For each of the following propositions, determine if it is a tautology, a contradiction or neither.

      i. $\neg(p \rightarrow \neg p)$

      ii. $(\neg p \vee q) \leftrightarrow (p \rightarrow q)$

      iii. $(p \wedge q) \vee (\neg p \wedge \neg q)$

2. (a) Write down the truth tables for $p \vee q$, $p \wedge q$ and $\neg p$.

   (b) Use the method of resolution to check if the following arguments are valid:

      i. $\{p \vee q, p \rightarrow s, q \rightarrow s\} \models s$.

      ii. $\{q, \neg p \leftrightarrow \neg q\} \models p$.

3. (a) Find the disjunctive normal form of the following propositions

      i. $\neg p \wedge (\neg q \rightarrow r)$.

      ii. $p \leftrightarrow (q \wedge r)$.

      iii. $\neg(p \rightarrow q)$.

   (b) Using natural deduction, show the following

      i. $\vdash (\neg p \rightarrow p) \rightarrow p$.

      ii. $\vdash (p \rightarrow q) \rightarrow (\neg q \rightarrow \neg p)$.

Section B

Answer only two questions

4. (a) Is there a DFA A such that $L(A) = \{a^n b^n \mid n \geq 0\}$? Justify your answer.

   (b) For each of the following languages, construct a DFA that recognises it.

   i. $L = \{a^n \mid n \equiv 1 \mod 3\}$, where the alphabet A is $\{a\}$.

   ii. $L = \{A^* baa A^*\}$, where the alphabet A is $\{a, b\}$.

5. (a) Construct an NFA with ϵ-transitions that recognises the language

$$(a^2 b^* + b^2 a^*)(ab + ba).$$

   (b) Suppose you are given NFAs with ϵ-transitions A_1 and A_2, that recognise the languages L_1 and L_2 respectively.

   i. Construct an NFA with ϵ-transitions that recognises $L_1 + L_2$

   ii. Construct an NFA with ϵ-transitions that recognises $L_1 L_2$

   iii. Construct an NFA with ϵ-transitions that recognises L_1^*

6. (a) Construct a Turing machine that recognises the language $L = \{a^n b^n \mid n \geq 0\}$

   (b) Construct Turing machines that compute $f(n)$ for

   i.

$$f(n) = n + 1$$

   ii.

$$f(n) = \begin{cases} 1, & n \leq 2 \\ 3, & n \geq 3 \end{cases}$$

   Show how your machines run on the tape $\ldots \sqcup 1111 \sqcup \ldots$, when the starting position of the head is the leftmost 1.