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OLLSCOIL NA hÉIREANN, GAILLIMH
THE NATIONAL UNIVERSITY OF IRELAND, GALWAY

SUMMER EXAMINATIONS 2004 - HONOURS

FINANCIAL MATHEMATICS AND ECONOMICS

MA111 - MATHEMATICS OF FINANCE

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Time allowed: *Three* hours.
Full marks for *six* questions.

1. (a) 1,000 euros is deposited on 3^{rd} April at 7% simple interest and is withdrawn on 7^{th} June.

Find the interest earned on each of the following bases:

(i) the exact length of time as a proportion of a (non-leap) year (Exact Simple Interest)

(ii) ordinary simple interest

(iii) the Banker's Rule.

(b) How long will it take money to double at 10% simple interest?

(c) At what simple interest rate will it take two years for 100 euros to accumulate to 120 euros?

p.t.o.

2. (a) Show that if the (compound) interest rate i corresponds to the (compound) discount rate d, then

$$i - d = id.$$

Find the discount rate which corresponds to an interest rate of 8%.

(b) Find the amount which must be invested now to accumulate to 1,000 euros at the end of three years, on each of the following bases:
(i) using simple interest at 6%
(ii) using simple discount at 6%
(iii) using compound interest at 6%
(iv) using compound discount at 6%

3. (a) What is the effective interest rate if money is worth 6%, compounded
(i) monthly
(ii) continuously.

(b) Find the nominal interest rate, compounded quarterly which is equivalent to 8% compounded semi-annually.

(c) Which of the following gives the best return on an investment and which gives the least return on an investment, where the term of the investment is greater than one year?

(i) 12% interest compounded monthly
(ii) 12% effective interest
(iii) 12% force of interest

p.t.o.

4. (a) Suppose you have debts of S_1 euros to be paid in t_1 years and S_2 euros to be paid in t_2 years. If you wish instead to pay off these debts with a single payment of S euros in t years, show that

$$Sv^t = S_1 v^{t_1} + S_2 v^{t_2}$$

where $v = \frac{1}{1+i}$.

(b) Find the amount of the single payment you should make in two years to settle debts of 100 euros due in four years and 500 euros due in six years, if the interest rate is 3%.

(c) Use the Method of Equated Time to estimate the time at which a single payment of 600 euros will settle debts of 100 euros due in four years and 500 euros due in six years.

5. (a) Suppose that 1 euro is invested at r% interest.

(i) Derive a formula for the time t it takes for this money to double.

(ii) Describe the Rule of 72 and use it to estimate the doubling time if $r = 6\%$.

(b) Suppose you borrowed 1,000 euros from a loan-shark two years ago at 30% interest convertible quarterly and you wish to repay this debt by borrowing from a bank at 5% effective interest.

If the term of the bank loan is four years, how much will you have to repay the bank at the end of four years?

p.t.o.

6. (a) Let the interest period = payment period and let i be the interest rate per conversion period. Show that the present value of a deferred annuity of €1 made at the end of each period for n equal periods with k deferred payment intervals is

$$\frac{1-(1+i)^{-n}}{i}\,(1+i)^{-k}.$$

(b) What sum of money should be set aside at Susan's birth to provide 8 semi annual payments of €10,000 for Susan's university education if the first payment is to be made on Susan's 17th birthday if we assume that the fund earns interest at 4% compounded semi-annually?

(c) An ordinary annuity compounded monthly at 12% pays €150 per month for 2 years and €250 per month for the next year. Find the accumulated value of the annuity.

7. (a) Let c be the number of interest periods in one payment period and i the interest rate per conversion period. Show that the accumulated value of a complex annuity due of €1 is

$$s_{\overline{nc}|i}\,\frac{1}{a_{\overline{c}|i}}.$$

(b) The present value of an ordinary annuity of €100 payable at the end of every six months for 15 years is €2,300. Use the tables and linear interpolation to find the nominal rate of interest compounded semi-annually.

p.t.o.

8. (a) An ordinary annuity compounded quarterly at 8%, pays €100 semiannually for 20 years and €300 semi annually for 10 years after this. Find the present value of the annuity.

(b) A leaves an estate of €100,000. Interest on the estate is paid to beneficiary B for the first 7 years, to beneficiary C for the next 20 years and to charity D thereafter. Find the relative shares of B, C and D in the estate, if it is assumed that the estate will earn a 2% effective rate of return.

9. (a) Susan's house painted 8 years ago for €10,000 now needs to be repainted. If Susan decides to switch to a better paint from now on she will only need to paint her house every 10 years. How much can she afford to pay for the better paint if the capitalised cost remains the same, given that the effective rate of interest is 6%.

(b) Let €P be the outstanding loan balance of a debt €A being amortised by equal payments of €R over n periods at rate i per period. Determine P (after k periods) both by the prospective and by retrespective methods, and show the formulas are equal.

10. (a) Let the interest period = payment period and let i be the interest rate per conversion period. Show that the present value of a simple perpetuity due of €1 is

$$\frac{1+i}{i}$$

(b) A loan of €15,000 is being repaid by payments of €1,000 at the end of each month for as long as necessary, plus a smaller final payment. Interest charged is 12% compounded monthly. Find

(i) the number of payments that will be necessary

(ii) the amount of the final payment

(iii) the outstanding loan balance after the 5^{th} payment, and

(iv) the interest paid and the amount the principal is reduced in the 6^{th} payment.