

OLLSCOIL NA hÉIREANN, GAILLIMH
THE NATIONAL UNIVERSITY OF IRELAND, GALWAY

SUMMER EXAMINATIONS 2004

FIRST ARTS EXAMINATION

MA121 - CALCULUS

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Time allowed: *three* hours.

Answer *five* questions.

1. Evaluate the following limits:

(a) (i) $\lim_{x \rightarrow -1} \frac{x^2 + 3x + 2}{x + 1}$

(ii) $\lim_{\theta \rightarrow 0} \frac{\sin(3\theta)}{\tan(4\theta)}$

(b) Determine the horizontal and vertical asymptotes of:

$$f(x) = \frac{5 - 4x + x^2}{2 - 3x + x^2}$$

(c) Differentiate with respect to x : $e^{5x} \frac{\cos(x)}{\sin(x)}$

(d) Use logarithmic differentiation to determine $\frac{dy}{dx}$ when

$$y = \frac{x^2 e^x}{x + 2}$$

P.T.O.

2. Let $f(x) = x^3 + 3x^2 - 9x + 7$.

(a) Determine the intervals on which f is:

(i) increasing; (ii) decreasing; (iii) concave up; (iv) concave down.

(b) Show that f has a local maximum at $x = -3$, and determine the local minimum and the point of inflection.

(c) Determine the absolute maximum and the absolute minimum on the closed interval $[0, 3]$.

3. (a) The U.S. postal service will accept a box for domestic shipment only if the sum of the length and the base perimeter does not exceed 108in. Find the dimensions of the largest acceptable box with a square end.

(b) The volume of a melting snowball is decreasing at a rate of 3cm^3 per minute. At what rate is the radius changing when the radius is 2cm ?

4. (a) Evaluate

$$\int_{-3}^1 |2x + 4| dx .$$

(b) Calculate the area of the finite region bounded by the curves $y = x^2 - x - 2$ and $y = 2 + x - x^2$.

(c) Water is pumped into an initially empty 400-litre tank at a rate of $40 - 2t$ litres per minute at time t minutes. How much water is in the tank after 10 minutes? How long will it take to fill the tank?

5. Calculate three of the following:

(a) $\int \frac{x^3}{(x-1)^2} dx$

(b) $\int x^2 \sin(x) dx$

(c) $\int \frac{x^3}{\sqrt{1-x^2}} dx \quad |x| < 1$

(d) $\int \frac{x^2}{(x-2)^2(x+2)} dx$

6. (a) Define the natural logarithm function $\ln(x)$ as an integral. Define the exponential function $\exp(x)$. Then prove that

$$\ln(xy) = \ln(x) + \ln(y) \quad (x, y > 0),$$

$$\frac{d}{dx} \exp(x) = \exp(x).$$

- (b) The area bounded by the curves $y = x$ and line $y = x^2$ is revolved about the x -axis. Calculate the volume of the resulting solid.

7. (a) Solve the following differential equation.

$$x \frac{dy}{dx} - 2y = x^3, \quad y(1) = 0.$$

- (b) A 200-litre tank initially full of water develops a leak at the bottom, and 20% of the water leaks in the first 10 minutes. Find the amount of water in the tank after 20 minutes if the water drains at a rate proportional to the product of the time elapsed and the amount of water present.