

OLLSCOIL NA hÉIREANN, GAILLIMH  
NATIONAL UNIVERSITY OF IRELAND, GALWAY

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SUMMER EXAMINATIONS 2004

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SECOND UNIVERSITY EXAMINATION

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MATHEMATICS  
MA203 – LINEAR ALGEBRA

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Prof. T. Hurley  
Prof. M. Newell

Time allowed: **Two** hours.  
Attempt **three** questions.

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1. (a) For the following homogeneous system of linear equations:

$$\begin{array}{rcccccccl} 2x_1 & - & 2x_2 & + & x_3 & + & x_5 & & = & 0 \\ -x_1 & + & x_2 & - & 2x_3 & + & 3x_4 & + & x_5 & = & 0 \\ x_1 & - & x_2 & + & 2x_3 & & & - & x_5 & = & 0 \\ & & & & -x_3 & - & x_4 & + & x_5 & = & 0 \end{array}$$

- (b) Show two of the variables can be chosen arbitrarily.  
(c) Find the general solution of the system.

p.t.o.

2. (a) Let  $A = \begin{pmatrix} 1 & 1 & 0 & 1 \\ 2 & 0 & 2 & 2 \\ -1 & 0 & 2 & 1 \\ 2 & 1 & 0 & -1 \end{pmatrix}$

Using Gauss-Jordan reduction find:

- (i) the determinant  $|A|$  of  $A$ ,
- (ii) the last row of  $A^{-1}$ .

3. (a) Let  $A = \begin{pmatrix} 1 & 2 & -1 \\ 1 & 0 & 1 \\ 4 & -4 & 5 \end{pmatrix}$ .

- (i) Find the eigenvalues and corresponding eigenvectors of  $A$ .
- (ii) Find a matrix  $E$  such that  $E^{-1}AE$  is diagonal.

4. (a) Let  $M$  be the transition matrix of a Markov process. Explain why 1 is always an eigenvalue of  $M$ .
- (b) An airline has planes based in Galway, Knock and Shannon. Each week  $\frac{1}{4}$  of the planes originally based in Galway are based in Knock and  $\frac{1}{3}$  of the planes based in Galway end up in Shannon - the remainder return to the Galway base. Of the Knock planes  $\frac{1}{5}$  end up at the Galway base and  $\frac{1}{10}$  at the Shannon base. The remainder return to Knock. Finally, of the Shannon planes  $\frac{1}{5}$  are based in Galway and  $\frac{1}{5}$  are based in Knock at the end of the week, the remainder based in Shannon.

Find the steady state of this Markov process.