

OLLSCOIL NA hÉIREANN, GAILLIMH
NATIONAL UNIVERSITY OF IRELAND, GALWAY

SUMMER EXAMINATIONS 2004

SECOND UNIVERSITY (Arts and Science) EXAMINATION

MATHEMATICS [MA212]

PASS

Dr. Dave Johnson

Professor T.C. Hurley

Dr. P. Kirwan

Time Allowed: *Two* hours

Full marks for *three* questions

Question 1

(a) Let $z = 2xe^{5y}$. Show that

$$\frac{\partial z}{\partial y} - 4x \frac{\partial z}{\partial x} = z$$

(b) Let $z = \frac{1}{2} \ln(x^2 + y^2)$.

(i) Evaluate the partial derivatives $\frac{\partial z}{\partial x}$, $\frac{\partial z}{\partial y}$, $\frac{\partial^2 z}{\partial x^2}$, $\frac{\partial^2 z}{\partial y^2}$ and $\frac{\partial^2 z}{\partial x \partial y}$.

(ii) Show that $x \frac{\partial z}{\partial x} + y \frac{\partial z}{\partial y} = 1$.

(iii) Show that $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0$.

P.T.O.

Question 2

- (a) Let S be the surface $x^2 + 4y^2 = z^2$.
- (i) Determine the equation for the tangent plane to the surface S at the point $(5, 6, 13)$.
- (ii) Determine the parametric equations for the normal line to the surface S at the point $(3, -2, 5)$.
- (b) Use the cross product to determine parametric equations for the line in which the following planes meet.

$$\begin{aligned}x - 2y + 4z &= 4 \\x + y - 2z &= 5\end{aligned}$$

Question 3

- (a) For the function $f(x, y) = x^2y + 2y^2x$ at the point $(2, -1)$, determine;
- (i) the direction of greatest increase in f ,
- (ii) the derivative of f in the direction of greatest increase in f ,
- (iii) the directions in which the derivative of f is zero.
- (b) Find the stationary points of the function

$$f(x, y) = 2xy - 5x^2 - 2y^2 + 4x - 4$$

and determine if they are local maxima, minima or saddle points.

Question 4

- (a) Let $w = x^2e^y, x = u + v, y = u - v$.

- (i) Use the chain rule to evaluate the derivatives $\frac{\partial w}{\partial u}$ and $\frac{\partial w}{\partial v}$.
- (ii) Verify that

$$\left(\frac{\partial w}{\partial u}\right)\left(\frac{\partial w}{\partial v}\right) = \left(\frac{2w}{x}\right)^2 - w^2$$

- (b) Determine the points on the sphere $x^2 + y^2 + z^2 = 16$ where

$$f(x, y, z) = 2x + y + z$$

has its maximum and minimum values.