GX2034

OLLSCOIL NA hÉIREANN, GAILLIMH
NATIONAL UNIVERSITY OF IRELAND, GALWAY

SUMMER EXAMINATIONS 2004

THIRD UNIVERSITY (Arts and Science) EXAMINATION

MATHEMATICS [MA300]

MA302 – Complex Variables

PASS

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Time Allowed: *Two* hours
Third Arts Mathematical Studies: Full marks for *four* questions.
All other students: Full marks for *three* questions

Question 1

(a) **(i)** Let

$$f(z) = \begin{cases} \dfrac{z^2+1}{z-i} & z \neq i \\ 1+2i & z = i \end{cases}$$

Is f continuous at the point $z_0 = i$?

(ii) Use l'Hôpital's rule to determine $\lim_{z \to 5i} \dfrac{z^2 - 3iz + 10}{z^2 + 25}$.

(b) Suppose that

$$f(z) = 8x - x^3 - xy^2 + i(x^2y + y^3 - 8y)$$

where $z = x + iy$. Determine where $f'(z)$ exists and calculate its value there.

(c) Show that $u(x,y) = xy^3 - x^3y$ is harmonic and determine a harmonic conjugate v of u. Write $u + iv$ as a function of the complex variable $z = x + iy$.

P.T.O.

Question 2

(a) Determine all complex values z with $e^{2iz-i} = -i$.

(b) Evaluate: $\int_C |z^2|dz$ where C is the curve from $z = 0$ to $z = 1 + i$ given by the parameterization $z(t) = t + it^2$ for $0 \leq t \leq 1$.

(c) Show that $\sinh^{-1}(z) = \log(z + (z^2+1)^{\frac{1}{2}})$ and determine all values of $\sinh^{-1}(-i)$.

Question 3

(a) State Cauchy's integral formula and use it to evaluate

$$\int_C \frac{1}{1+z^2}dz$$

where C is the positively oriented circle of radius 1 centered at
(i) $z = i$; (ii) $z = -i$.

(b) Determine the Laurent series about 0 for

$$f(z) = \frac{5-z}{z^2-z-2}$$

valid in (i) $1 < |z| < 2$; (ii) $|z| > 2$.

Question 4

(a) Define the following terms:

(i) isolated singular point;

(ii) a pole of order m.

(b) Determine the residues at each pole of the function

$$f(z) = \frac{6z-3}{(z^2-4iz-4)(z+1)}$$

(c) Use the Residue Theorem to evaluate the integral

$$\int_C \frac{6z-3}{(z^2-4iz-4)(z+1)}dz$$

where C is the circle of radius 2 centered at $z = -1 - i$.