GX2036

OLLSCOIL NA hÉIREANN, GAILLIMH
NATIONAL UNIVERSITY OF IRELAND, GALWAY

SUMMER EXAMINATIONS, 2004

THIRD UNIVERSITY EXAMINATION

MA310 - ACTUARIAL MATHEMATICS 1

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Dr. M. Hayes

Time allowed: *Two* hours.

Marks shown for each question.

In addition to this paper you should have available actuarial tables and an electronic calculator which is not capable of storing text.

PART A: ANSWER TWO OUT OF THREE QUESTIONS.

1. If the effective rate of discount convertible every month is 10% p.a., calculate the nominal rate of interest p.a. convertible every 6 months and the force of interest.

[5]

2. Show that

$$(Da)_{n\rceil} = \frac{n - a_{n\rceil}}{i}.$$

[5]

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3. Find the accumulated value of a continuously payable annuity $\rho(t)$ where

$$\rho(t) = t,$$

over the last year if the force of interest is 1% p.a.

[5]

PART B: ANSWER TWO OUT OF THREE QUESTIONS.

4. A project has an initial outlay on 1/5/2004 of €1,000,000. Six months later a further expenditure of €500,000 will be required. On 1/5/2005 income will be received of €50,000 per quarter payable in advance for 30 years. The project will be complete when the income stops. The income increases by 5% per annum compound on 1st May each year starting in 2020.

Calculate the accumulated profit of the project at a rate of 5% per annum.

[10]

pto

5. A company who has just been awarded a contract has the following two options: Option A - do the work in house, Option B - contract out the work.

Option A

Description	Timing	Amount (€'000's)
Equipment	start	−425
Staff costs	during year 1	−85
Staff costs	during year 2	−90
sales	end year 2	750

Option B

Description	Timing	Amount (€'000's)
Contractors fees	start year 1	−300
Contractors fees	start year 2	−310
sales	end year 2	750

(a) Calculate the present value and accumulated profit of each option with a risk discount rate of 20 % p.a.

(b) Find the yield to redemption of each option.

(c) What is the best option for the company? Explain your answer.

[10]

pto

6. The following are total valuations of a insurance fund (€000's).

Year	1st Jan	1st April	1st July	1st October
2002	1010	1210	1100	1050
2003	1100	1200	1150	1199
2004	1400			

On 30th April, 2002, €40,000 worth of Irish fixed interest securities were bought. On 31st December, 2002 interest on cash of €20,000 were received together with a premium investment of €200,000. Interest on cash of €40,000 was received on 30th June, 2003. Investment managers were paid €20,000 from the fund on 31th December, in 2002 and 2003.

(a) Calculate the money weighted rate of return p.a. in each quarter of 2001 and 2002.

(b) Calculate the time weighted rate of return p.a. for the two year period ending 31st December 2003.

(c) Calculate the quarterly linked internal rates of return p.a. for the years 2002 and 2003.

[10]

PART C: ANSWER TWO OUT OF THREE QUESTIONS.

7. A fixed interest stock pays a coupon of 8% per annum twice yearly. The next coupon is due on 1st July, 2004. It is redeemable at 110% at the option of the borrower on any 1st January between 2030 and 2035 inclusive.

Calculate the price an investor would pay on 30th June, 2004 to receive an effective net yield of at least 10% per annum. The investor pays tax at 33% on income received. Tax payments on coupons are due four months after each coupon is paid. Capital gains tax is paid at 34% on 15th November on any capital gain realized during the previous 12 months to 24 months.

[17.5]

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8. An investor who is taxed at 40%, purchased 1000 shares on 30th June, 2003, in a company. All income tax is paid on 30th September for all income received over the previous 12 months. Dividends are paid on the same date annually and the last dividend of €10 per share was paid 6 months before the date of purchase. The dividends are expected to rise by 4% each calendar year. Calculate the price paid by the investor if the expected net yield is 10% p.a. effective.

[17.5]

9. The rent for the next ten years on a property contract is set at €5,000 per month in arrears. The rent will increase by 20 % compound every ten years. Property rental tax is at a rate of 30 % and is paid every two years for all rent received over the previous two years. Find the price of the property so as a new investor will achieve an investment return of 8 %p.a. effective.

[17.5]

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PART D: ANSWER ONE OUT OF ONE QUESTION.

10. (a) Define and give two examples of 'an arbitrage'.

(b) Explain clearly the assumption of ' no arbitrage' in financial mathematics.

(c) A fixed interest security pays coupons of 8% p.a. which are payable half-yearly in arrear and is redeemable at 110%. Four months before the next coupon is due, an investor negotiates a forward contract in which he agrees to buy €40,000 nominal in 1 years time. The current price of the stock is €90 per €100 nominal and the effective rate of interest is 4% p.a. Calculate the forward price.

[15]

PART E: ANSWER TWO OUT OF THREE QUESTIONS.

11. (a) Calculate the price of a five year fixed interest security redeemable at par with 4% annual coupons given the following information:

$$\begin{aligned} p_1 &= 90\% \\ y_2 &= 5\% \\ Y_3 &= 7\% \\ F_{3,1} &= 4\% \\ f_{3,2} &= 10\% \end{aligned}$$

(b) The prices of zero coupon bonds for various terms are as follows:

$5] = €85\%$ $10] = €70\%$ $15] = €30\%$

Calculate Y_{15} and $F_{10,5}$

[10]

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12. (a) Find the price at issue of a unit zero coupon bond with a 3 year term if the continuous time spot rates are :

$$Y_1 = 7\%, \ Y_2 = 8\%, \ Y_3 = 10\%.$$

(b) Calculate the gross redemption yield on a 4 year fixed interest security redeemable at 120% if the annual coupon is 3%, given

$$Y_1 = 7\%, \ p_2 = 80\%, \ y_3 = 9\%, \ F_{3,1} = 10\%.$$

[10]

13. (a) Show that for a small movement ϵ in interest rates from i to $i + \epsilon$ the change in the relative present value of cash flow is $-\epsilon\nu(i)$ where $\nu(i)$ is the effective duration.

(b) Calculate the duration for a bond redeemable at par in 3 years time with annual coupons of 3% at an interest rate of 4%.

[10]