GX2037

OLLSCOIL NA hÉIREANN, GAILLIMH

NATIONAL UNIVERSITY OF IRELAND, GALWAY

SUMMER EXAMINATIONS 2004

THIRD UNIVERSITY EXAMINATION

MA314 – LINEAR ALGEBRA

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Time allowed: **Two** hours.

Third Arts Mathematical Studies: Full marks for 4 questions.

All other students: Full marks for 3 questions.

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1. (a) Find an orthonormal basis for the subspace of $\mathbb{R}^3$ consisting of solutions to the equation

$$x + y + 2z = 0$$

(b) Find the point in the plane $x + y + 2z = 0$ that is closest to the point $(1, -2, 1)$.

(c) Suppose that $\{u_1, u_2\}$ is an orthonormal basis for $\mathbb{R}^2$ and that v is any vector in $\mathbb{R}^2$. Show that

$$v = (v.u_1)u_1 + (v.u_2)u_2.$$

2. (a) Find the least squares approximation of the form $y = ax^2 + bx$ to the data

$$\begin{aligned} (x_1, y_1) &= (-1, 1) \\ (x_2, y_2) &= (0, 0) \\ (x_3, y_3) &= (1, 2) \end{aligned}$$

(b) For continuous functions $f : [0,1] \longrightarrow \mathbb{R}$ and $g : [0,1] \longrightarrow \mathbb{R}$, let

$$\langle f, g \rangle = \int_0^1 f(t)g(t)\,dt.$$

Show that $\langle , \rangle$ is an inner product on the space of continuous functions $[0,1] \longrightarrow \mathbb{R}$.

3. (a) Let A be a symmetric matrix and let $\mathbf{v}_1$ and $\mathbf{v}_2$ be eigenvectors corresponding to distinct eigenvalues. Show that

$$\mathbf{v}_1^T \mathbf{v}_2 = 0$$

(b) Find the principal axes of the curve

$$x^2 - xy + y^2 = 1$$

and hence sketch the curve.

4. Use the simplex method to maximise the expression $x + 2y + z$ subject to the constraints

$$\begin{aligned} 2x + y + z &\leq 5 \\ x + 3y + 2z &\leq 10 \\ x, y, z &\geq 0 \end{aligned}$$