

OLLSCOIL NA hÉIREANN, GAILLIMH
NATIONAL UNIVERSITY OF IRELAND, GALWAY

SUMMER EXAMINATIONS 2004

B.Sc. (Part I) and B.A EXAMINATION

MATHEMATICS
MA342 – TOPOLOGY
HONOURS

Dr. Dave Johnson
Prof. T. Hurley
Dr. J. Cruickshank

Time allowed: **Two** hours.

Attempt **one** question from Section A, **one** question from Section B and
one question from Section C.

Section A

1. (a) What is a topological space?
- (b) Let X be a nonempty set and let

$$\mathcal{T} = \{F \subset X \text{ such that } |F| < \infty\} \cup \{X\}.$$

Show that \mathcal{T} is a topology on X .

- (c) What is a connected topological space?
- (d) Let X be a set with at least 2 distinct elements and let \mathcal{T} be the topology described in part (b). Show that the topological space (X, \mathcal{T}) is connected if and only if the set X is infinite.

2. (a) What is a loop in a space X ?
- (b) Which of the following spaces is pathwise connected? Justify your answer in each case.
 - i. $\mathbb{R}^2 - \{(0,0)\}$
 - ii. $\{(x,y,z) \in \mathbb{R}^3 : x + y + z \neq 0\}$
- (c) What does it mean to say that two paths α and β are homotopic relative to their endpoints?
- (d) Write down an example of a loop in \mathbb{R}^2 that has winding number 2 with respect to the point $(0,0)$.

Section B

3. (a) Let X and Y be metric spaces and let $f : X \rightarrow Y$ be a continuous function. Show that for every closed subset $A \subset Y$, $f^{-1}(A)$ is a closed subset of X .
- (b) Use the intermediate value theorem to show that $[0,1]$ is a connected space.
4. (a) Describe the two person cake cutting problem.
- (b) What assumptions will ensure that the two person cake cutting problem has a solution? Explain your answer.

Section C

5. Let A be a bounded closed subset of \mathbb{R}^n . Show that A is sequentially compact.
6. State and prove the Fundamental Theorem of Algebra. You should state clearly any lemmas or propositions that you use without proof.