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OLLSCOIL NA hÉIREANN, GAILLIMH
NATIONAL UNIVERSITY OF IRELAND, GALWAY

SUMMER EXAMINATIONS 2004

B.Sc. EXAMINATION

FOURTH YEAR MATHEMATICS OPTION

[MA426]

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Time allowed: *Two* hours.
Full marks for *three* questions.

1. The mixed strategies $\mathbf{r}$ and $\mathbf{s}$ are optimal for the players R and C in the zero-sum game defined by the matrix A, and $v_R \leq E(\mathbf{r}, \mathbf{s}) \leq v_C$. Explain the terms and symbols here, justify the inequalities and state von Neumann's theorem.

   Now suppose that $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$. Show that there is no saddle-point if and only if each of a and d is greater than, or less than, each of b and c. Prove von Neumann's theorem in the 2×2 case.

   Solve the game defined by the following matrix, taking care to explain and justify your solution.

$$A = \begin{pmatrix} 1 & 2 & -5 \\ -3 & -4 & 2 \\ -2 & 1 & 3 \end{pmatrix}.$$

2. (i) Describe the tableau (simplex) method for solving a pair of dual LPPs. Illustrate the method starting from this tableau:

$$
\begin{array}{rrr|rrr|r}
P_1 & P_2 & P_3 & & & & \\
4 & 1 & 13 & 1 & 0 & 0 & 1 \\
4 & 2 & 12 & 0 & 1 & 0 & 1 \\
4 & 13 & 2 & 0 & 0 & 1 & 1 \\
\hline
-1 & -1 & -1 & 0 & 0 & 0 & 0
\end{array}
$$

(ii) Hence solve the game defined by

$$
A = \begin{pmatrix} 1 & -2 & 10 \\ 1 & -1 & 9 \\ 1 & 10 & -1 \end{pmatrix}
$$

3. Describe and justify methods for finding every Nash equilibrium (NE) of a (non-cooperative) non-zero-sum game defined by 2×2 matrices A and B. Use the methods to find all the NEs of this instance of Chicken:

$$
A = \begin{pmatrix} 0 & -2 \\ 1 & -8 \end{pmatrix}, \quad B = A^t = \begin{pmatrix} 0 & 1 \\ -2 & -8 \end{pmatrix}
$$

Which of these NEs is Pareto optimal?

What is the best outcome of the game if the two players use the same (mixed) strategy?

Give the definition of an evolutionarily stable strategy (ESS) in a symmetric game. Determine the unique ESS in the game above.

4. (a) What are the payoff polygon, the status quo point and the bargaining (negotiation) set for a cooperative non-zero-sum game? Describe the Nash scheme to find an arbitration point for such a game, and use it to solve the game

$$A = \begin{pmatrix} 0 & 6 \\ 3 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 2 \\ 9 & 0 \end{pmatrix}, \quad SQ = (1,1).$$

(b) State (without proof) how to find the arbitration point when the bargaining set is a line segment with slope -1. How does this influence the choice of the status quo point? Solve the game

$$A = \begin{pmatrix} 0 & 7 \\ 2 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 2 \\ 7 & 0 \end{pmatrix}.$$