Ollscoil na hÉireann, Gaillimh
National University of Ireland, Galway

GX 2142

Christmas Examinations, 2003/2004

Exam Code(s)	2IT1
Exam(s)	SECOND INFORMATION TECHNOLOGY
Module Code(s)	MP209
Module(s)	**Methods of Mathematical Physics I**
Paper No.	
Repeat Paper	
Special Paper	
External Examiner(s)	Professor B. Straughan;
Internal Examiner(s)	Dr. M.S. Ó Confhaola;
	Dr. B. Gleeson.
__Instructions__:	No more than *THREE* questions to be attempted.
Duration	*TWO* hours
No. of Answer books	________
__Requirements__:	________
Handout	________
MCQ	________
Statistical Tables	YES, LOG TABLES
Graph Paper	________
Log Graph Paper	________
Other Material	________
No. of Pages	2 PAGES (Excluding Front Page)
Department(s)	MATHEMATICAL PHYSICS

1. (a) Show that the following pair of functions

$$u(x,y) = y^2 - x^2 \quad \text{AND} \quad v(x,y) = -2xy$$

satisfy the pair of partial differential equations

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \quad \text{AND} \quad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}.$$

Hence or otherwise, show that these functions also satisfy

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \quad \text{AND} \quad \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} = 0.$$

(b) If the cartesian coordinates x and y are related to the plane polar coordinates r and θ by $x = r\cos(\theta)$ and $y = r\sin(\theta)$, prove that

$$\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2 = \left(\frac{\partial f}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial f}{\partial \theta}\right)^2.$$

2. (a) Find all of the stationary points of the function

$$f(x,y) = x^3 + y^3 + 3x^2 + 6y^2 - 9x + 9y + 1,$$

and investigate the nature of each of these points.

(b) Use the method of Lagrange Multipliers to find the points on the curve, described by

$$x^2 + 8xy + 7y^2 = 180$$

which are closest to, and/or farthest from the origin $(0,0)$.

3. The periodic function $f(x)$ is defined, on the interval $-\pi \leqslant x < \pi$, by

$$f(x) = x,$$

and elsewhere by $f(x + 2\pi) = f(x)$.

(a) Find the Fourier series of $f(x)$.

(b) Sketch the function $f(x)$ on the interval $-3\pi \leqslant x < 3\pi$.

(c) State the value to which the series converges at $x = \pi/2$. Hence prove that:

$$\frac{\pi}{4} = \frac{1}{1} - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots$$

4. (a) Find the area of the region (denoted by R) bounded by the parabolas $y = x^2 - 32$ and $y = 18 - x^2$ by evaluating the area integral:

$$\mathrm{A} = \iint_{\mathrm{R}} dydx.$$

   (b) Consider the integral

$$\int_0^2 \int_{x^2}^4 (x^2 + 2y)\, dydx.$$

   Sketch the region of integration. Change the order of integration of this integral, and hence evaluate the integral.

   (c) Use plane polar coordinates to evaluate the integral

$$\iint_{\mathrm{R}} 16xy\,(x^2 + y^2)\, dxdy$$

   where R is the disc described by $x^2 + y^2 \leqslant 4$.

5. Green's theorem (in the plane) states that

$$\iint_{\mathrm{R}} \left[\frac{\partial g}{\partial x} - \frac{\partial f}{\partial y}\right] dA = \oint_{\mathrm{C}} \left[f\,(x,y)\, dx + g\,(x,y)\, dy\right].$$

Verify Green's theorem for $f\,(x,y) = x + y$, $g\,(x,y) = y$ when R is the region enclosed by the curves $y = 1$, $y = x^2 - x - 5$, and C is its perimeter.