Ollscoil na hÉireann, Gaillimh GX 2146
National University of Ireland, Galway

Semester I Examinations, 2003/2004

Exam Code(s)	2CS1;2EL1;2PT1;2BS1
Exam(s)	Second Science
Module Code(s)	MP212
Module(s)	Mechanics
Paper No	1
Repeat Paper	
Special Paper	
External Examiner(s)	Professor Brian Straughan
Internal Examiner(s)	Dr. M. Ó Confhaola
	Professor M. F. McCarthy

Instructions:

Answer: THREE questions.
All questions carry the same marks.
A compendium of useful formulae is attached.

Duration	2 hrs
No. of Answer books	
Requirements	
Handout	
MCQ	
Statistical Tables	Yes - Log Tables
Graph paper	
Log Graph Paper	
Other Material	
No. of Pages	3
Department(s)	Mathematical Physics

1. A system of distributed forces and couples consists of three forces and two couples as follows:

   Forces $F_1 = 2i - 5j + 6k$ acting at the point $(2, 3, 1)$, $F_2 = 4i - 2j - 4k$ acting at the point $(0, -1, 3)$ and $F_3 = 5i + 4j + k$ acting at the point $(-1, 2, 0)$and couples with torque vectors C_1=-$6i - 7j$, $C_2 = 4j - 5k$. Find, and state clearly, the resultant of this system at the origin. Find, and state clearly the wrench resultant of the system.

2. P is a point a distance h vertically above the corner A of a horizontal square platform $ABCD$, of side ℓ, which is hinged to support along AB. The platform is held in an inclined position $ABC'D'$ by a rope, of length L, connecting P to the corner C'(diagonally opposite A). If the weight of the platform is W, show that the tension in the string is given by

$$T = \frac{WL}{2h}.$$

3. The left end of a uniform light cable is mounted $15m$ below the right end. The horizontal span between the support points is $100m$ and the sag, measured from the left side is $10m$.

   Find the maximum tension in the cable if the cable has a uniform horizontal loading distribution in the vertical direction of $2kNm^{-1}$

4. A particle of mass m moves in a one-dimensional space under the influence of a conservative force with potential energy

$$V(x) = c\frac{x - 2}{x^2 - 4x + 13}$$

   where c is a positive constant and x measures the position in one-dimensional space.

   Find the position of equilibrium of the particle and examine their stability properties.

   Sketch the energy diagram for the particle.

   If the particle passes through the position of equilibrium with speed u, find the values of u for which the particle must undergo oscillatory motion about the position of equilibrium.

5. A rocket of initial mass M_0, carrying a payload of mass P, is to be launched vertically from rest at the surface of the earth.

   (a) The rocket is initially designed as a single stage rocket with 90% of its initial mass as fuel. Show that, in the absence of gravity and other resisting forces, the maximum speed imparted by the rocket to the payload is

$$c \ln \left[\frac{P + M_0}{P + \frac{1}{10}M_0} \right]$$

   where c is the exhaust gas ejection speed.

   (b) If the rocket is re-designed as a two stage rocket, with the same initial mass M_0 and initial fuel content, but with the second stage being one third the mass of the first

stage, show that, in the absence of gravity and other resisting forces, the maximum speed attained by the payload is

$$c \ln \left[\frac{P + M_0}{P + \frac{13}{40} M_0} \cdot \frac{P + \frac{1}{4} M_0}{P + \frac{1}{40} M_0} \right].$$

Compute the two final velocities above when $c = 3kms^{-1}$ and the payload to initial mass ratio is $1:120$.

Useful Formulae

Force Systems

$$\mathbf{C}_A = \mathbf{C}_O - \mathbf{OA} \times \mathbf{F}$$

Chains and Cables

Equations for a light cable subject to a uniform load

$$T \cos\theta = H \text{ and } \frac{dy}{dx} = \frac{w}{H} x + C_1$$

Equations for a uniform heavy cable

$$\begin{aligned}
T \cos\theta &= H \text{ and } \frac{dy}{dx} = \frac{w}{H} s + C_1 \\
x &= \frac{H}{w} \sinh^{-1} \left(\frac{w}{H} s + C_1 \right) + D_2 \\
y &= \frac{H}{w} \cosh \frac{w}{H} (x - D_2) + C_2 \\
y &= \frac{H}{w} \sqrt{1 + \left(\frac{w}{H} s + C_1 \right)^2} + C_2
\end{aligned}$$

Rocket Motion

The one-dimensional variable mass equation of motion appropriate to the motion of rockets is

$$F_{tot}^{ext} = m(t) \frac{d}{dt} v(t) - \left(\frac{d}{dt} m(t) \right) v_{g\,|\,r}$$

where $m(t)$ is the mass of the rocket system at time t, $v(t)$ is the velocity of the rocket system at time t, and $v_{g\,|\,r}$ is the velocity of the exhaust gases relative to the rocket at time t.

Stabililty Theory

The equation $\ddot{x} = \frac{1}{m} F(x)$ possesses an equilibrium point at x_0 if $F(x_0) = 0$.
The equilibrium point $x = x_0$ is stable if $F'(x_0) < 0$, while it is unstable if $F'(x_0) > 0$.