Ollscoil na hÉireann, Gaillimh GX 2149
National University of Ireland, Galway

Semester I Examinations, 2003/2004

Exam Code(s)	3BS3,3BS4,3BS5,3BS9,3CS1,3CS2, 3EL1,3EL2,4BS3,4BS9,4CS2,1MF2
Exam(s)	Third Science, Fourth Science
Module Code(s)	MP305
Module(s)	Modelling I
Paper No	1
Repeat Paper	
External Examiner(s)	Professor Brian Straughan
Internal Examiner(s)	Dr. Mícheál Ó Confhaola
	Dr. M.Tuite

Instructions: Full marks for **THREE** correctly anwered questions.

Duration	2hrs
No. of Answer books	
Requirements	
Handout	
MCQ	
Statistical Tables	Yes - Log Tables
Graph paper	
Log Graph Paper	
Other Material	
No. of Pages	3
Department(s)	Mathematical Physics

1.

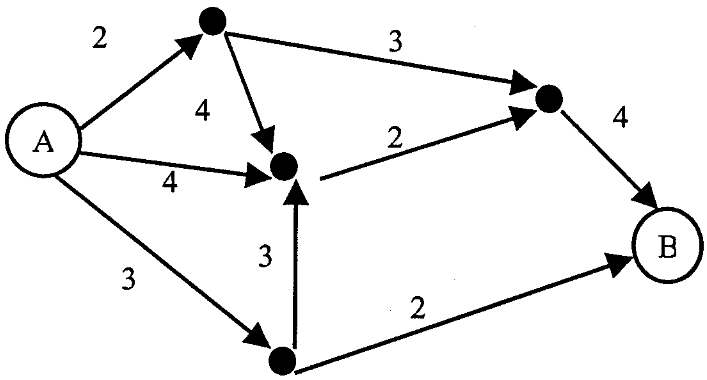
a. Describe what is meant by a cut $(P,\overline{P})$ of a network flow graph G. Define the capacity $c(P,\overline{P})$ and flow $\phi(P,\overline{P})$ of $(P,\overline{P})$ and show that

$$\phi(P,\overline{P}) \le c(P,\overline{P}),$$
$$\phi(P,\overline{P}) - \phi(\overline{P},P) = v,$$

where v is the network flow value. Hence show that if $v = c(P,\overline{P})$ for some cut $(P,\overline{P})$ then the maximal possible flow is attained.

b. Define the incremental network and describe the Ford-Fulkerson algorithm for finding the maximal integer flow through a network flow diagram. When and how does the algorithm terminate?

c. A road network is given below with road capacities shown. Find the maximal flow of cars from A to B and identify the cut with minimal capacity.



2.

a. Show that a flow through a network is of minimal cost for the given flow value if and only if the corresponding incremental network has no cycles of negative cost.

b. A jam factory buys units of fruit at the beginning of each month i at a purchase of p_i per unit. The firm can store up to 5 units of fruit every month. The consumption requirements are c_i units per month. Based on last year's figures the following estimates have been made for June, July and August:

i	June	July	Aug
p_i	2	3	1
c_i	1	2	3

Construct a network flow model with appropriate capacities and costs for this system. Find the best purchasing schedule assuming that the factory has no fruit in storage on June 1st.

3. A small pharmaceutical plant produces two compounds F and G from constituent compounds A, B, C, D and E all of which are manufactured at the plant. The constituent (if any) for each compound and the time taken by one worker to manufacture each compound is:

Compound	A	B	C	D	E	F	G
Constituents			A	A,B	A,B	C,D	D,E
Time	2	2	3	2	3	5	3

a. Construct an activity network based on the given precedence relations and completion times.

b. Find the critical path for this network. Find the earliest and latest starting times for each activity and the float. What is the minimum possible completion time assuming sufficient workers?

c. Suppose that **two** workers are assigned to complete these activities. Apply the Critical Path Scheduling strategy to find an optimal scheduling for these two workers. Comment on your answer.

4.

a. Consider one-dimensional traffic flow along a road. Let $u(x,t)$ be the speed of the traffic, $\rho(x,t)$ the traffic density and $q(x,t) = \rho(x,t)u(x,t)$ the traffic flow. Assuming conservation of flow and that $u = u(\rho)$ show that ρ satisfies the following partial differential equation

$$\frac{\partial \rho}{\partial t} + q'(\rho)\frac{\partial \rho}{\partial x} = 0.$$

b. If u is linearly dependent on ρ and if the traffic density is initially

$$\rho(x,0) = \begin{cases} \frac{1}{4}\rho_{\max}, & x < 0 \\ 0, & x > 0 \end{cases}$$

Discuss and plot the density at all later times by analysing the characteristics.

5.

a. Consider a zero-sum two player matrix game with pure strategies A_i and B_j for $i = 1\ldots m$ and $j = 1\ldots n$ and with payoff matrix a_{ij}. Show that a saddle-point solution exists if and only if there exists a value V obeying

$$V = \min_{j=1\ldots n} \max_{i=1\ldots m} a_{ij} = \max_{i=1\ldots m} \min_{j=1\ldots n} a_{ij}.$$

b. Consider the matrix game with payoff matrix:

	B_1	B_2	B_3	B_4
A_1	1	2	4	1
A_2	0	−2	−1	−1

Show that a saddle point solution exists. Is this solution unique?

c. Consider the game with payoff matrix

	B_1	B_2	B_3	B_4
A_1	1	1	4	−2
A_2	0	−4	−3	2

Find the optimal mixed stategy for this game and verify that the average value of the game is $-\frac{2}{3}$