Ollscoil na hÉireann, Gaillimh GX 2151
National University of Ireland, Galway
Semester I Examinations, 2003/2004

Exam Code(s)	3CS1
Exam(s)	Third Science
Module Code(s)	MP363
Module(s)	Methods of Maths Physics I
Paper No	1
Repeat Paper	
Special Paper	
External Examiner(s)	Professor Brian Straughan
Internal Examiner(s)	Dr. M. Ó Confhaola
	Professor M. F. McCarthy

Instructions:

Attempt THREE questions.

Duration	2 hrs
No. of Answer books	
Requirements	
Handout	
MCQ	
Statistical Tables	Yes - Log Tables
Graph paper	
Log Graph Paper	
Other Material	
No. of Pages	3 4
Department(s)	Mathematical Physics

1. (a) Show that if $y_1(x)$ and $y_2(x)$ are two real valued differentiable functions defined in a real interval I whose Wronskian is non zero at least one point in I, then $y_1(x)$ and $y_2(x)$ are linearly independent in I.

(b) Determine which of the following pairs of functions are linearly independent in the given interval.

(i) $\sin ax, \cos(ax), (a \neq 0)$ in $(-\infty, \infty)$

(ii) $e^{ax} \sin bx, e^{ax} \cos bx, (a \neq 0, b \neq 0)$ in $(-\infty, \infty)$.

(iii) $x^m, x^n (m \neq n)$ in $(0, \infty)$

(iv) $\sin(x-2), \sin(x+2)$ in $(-\infty, \infty)$.

2. (a) In each of the following differential equations, find all the singular points and determine whether each on is regular or irregular

(i) $y'' - (x-3) = 0$

(ii) $x^2 y'' - y' + y = 0$

(iii) $(x^2 + x)y'' - (x^2 - 2)y' - (x+2)y = 0$

(iv) $x^2(1-x)y'' - 2y' + 2y = 0$

(b) Find the power series solution of the ordinary differential equation

$$\frac{d^2y}{dx^2} - x\frac{dy}{dx} + 2y = 0$$

which is valid in the region of the point $x = 0$.

3. Consider the following ordinary differential equation:

$$2x^2 y'' + 7x(x+1)y' - 3y = 0$$

(a) Seek a power series solution about the point $x = 0$ of the form

$$y = \sum_{r=0}^{\infty} a_r x^{r+s}$$

and show that the indices are given by $x = \frac{1}{2}$ and $s = -3$.

(b) Use the method of Frobenius to find two linearly independent solutions of the differential equation.

4. (a) Let C denote the line segment from $z = i$ to $z = 1$. By observing that, of all points on that line segment, the midpoint is closest to the origin, show that

$$\left| \int \frac{dz}{z^4} \right| \leq 4\sqrt{2}$$

without evaluating the integral.

(b) Let C be the circle $|z| = R(R > 1)$, described in a counterclockwise direction. Show that

$$\left| \int_C \frac{Log z}{z^2} dz \right| < 2\pi \left(\frac{\pi + \ln R}{R} \right)$$

and hence that the value of the integral approaches zero as R tends to infinity.

5. (a) Find the value of the integral

$$\int_C \frac{3z^3 + 2}{(z-1)(z^2+9)} dz$$

taken counterclockwise about the circle (a)$|z-2| = 2$;(b)$|z| = 4$.

(b) Evaluate the integral

$$\int_0^\infty \frac{(2x^2 - 1)}{x^4 + 5x^2 + 24}.$$