

GX 2152

Exam Code(s) 3FM2

Exam(s) Third Science

Module Code(s) MP391

Module(s)	Mathematical Modelling
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Paper No 1

Repeat Paper _____

External Examiner(s) Professor Brian Straughan

Internal Examiner(s) Dr. Mícheál Ó Confhaola

Dr. M.Tuite

Duration 2hrs

No. of Answer books

Requirements

Handout

MCQ

Statistical Tables	Yes - Log Tables
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Graph paper

Log Graph Paper

Other Material

No. of Pages	3
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Department(s) Mathematical Physics

1.

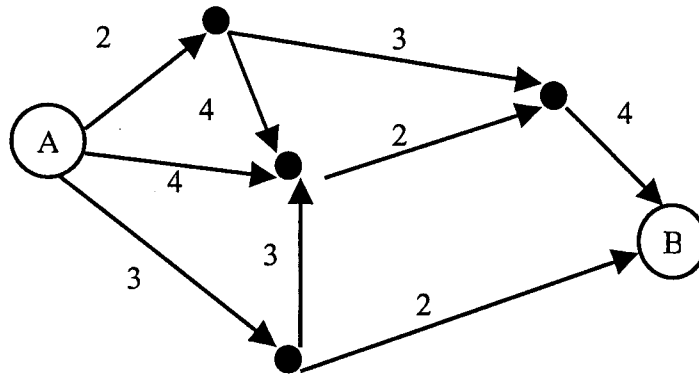
- a. Describe what is meant by a cut (P, \bar{P}) of a network flow graph G . Define the capacity $c(P, \bar{P})$ and flow $\phi(P, \bar{P})$ of (P, \bar{P}) and show that

$$\phi(P, \bar{P}) \leq c(P, \bar{P}),$$

$$\phi(P, \bar{P}) - \phi(\bar{P}, P) = v,$$

where v is the network flow value. Hence show that if $v = c(P, \bar{P})$ for some cut (P, \bar{P}) then the maximal possible flow is attained.

- b. Define the incremental network and describe the Ford-Fulkerson algorithm for finding the maximal integer flow through a network flow diagram. When and how does the algorithm terminate?
- c. A road network is given below with road capacities shown. Find the maximal flow of cars from A to B and identify the cut with minimal capacity.



2.

- a. Show that a flow through a network is of minimal cost for the given flow value if and only if the corresponding incremental network has no cycles of negative cost.
- b. A jam factory buys units of fruit at the beginning of each month i at a purchase of p_i per unit. The firm can store up to 5 units of fruit every month. The consumption requirements are c_i units per month. Based on last year's figures the following estimates have been made for June, July and August:

i	June	July	Aug
p_i	2	3	1
c_i	1	2	3

Construct a network flow model with appropriate capacities and costs for this system. Find the best purchasing schedule assuming that the factory has no fruit in storage on June 1st.

3. A small pharmaceutical plant produces two compounds F and G from constituent compounds A , B , C , D and E all of which are manufactured at the plant. The constituent (if any) for each compound and the time taken by one worker to manufacture each compound is:

Compound	A	B	C	D	E	F	G
Constituents			A	A, B	A, B	C, D	D, E
Time	2	2	3	2	3	5	3

- Construct an activity network based on the given precedence relations and completion times.
- Find the critical path for this network. Find the earliest and latest starting times for each activity and the float. What is the minimum possible completion time assuming sufficient workers?
- Suppose that **two** workers are assigned to complete these activities. Apply the Critical Path Scheduling strategy to find an optimal scheduling for these two workers. Comment on your answer.

4.

- Consider one-dimensional traffic flow along a road. Let $u(x, t)$ be the speed of the traffic, $\rho(x, t)$ the traffic density and $q(x, t) = \rho(x, t)u(x, t)$ the traffic flow. Assuming conservation of flow and that $u = u(\rho)$ show that ρ satisfies the following partial differential equation

$$\frac{\partial \rho}{\partial t} + q'(\rho) \frac{\partial \rho}{\partial x} = 0.$$

- If u is linearly dependent on ρ and if the traffic density is initially

$$\rho(x, 0) = \begin{cases} \frac{1}{4} \rho_{\max}, & x < 0 \\ 0, & x > 0 \end{cases}$$

Discuss and plot the density at all later times by analysing the characteristics.

5.

- a. Consider a zero-sum two player matrix game with pure strategies A_i and B_j for $i = 1 \dots m$ and $j = 1 \dots n$ and with payoff matrix a_{ij} . Show that a saddle-point solution exists if and only if there exists a value V obeying

$$V = \min_{j=1 \dots n} \max_{i=1 \dots m} a_{ij} = \max_{i=1 \dots m} \min_{j=1 \dots n} a_{ij}.$$

- b. Consider the matrix game with payoff matrix:

	B_1	B_2	B_3	B_4
A_1	1	2	4	1
A_2	0	-2	-1	-1

Show that a saddle point solution exists. Is this solution unique?

- c. Consider the game with payoff matrix

	B_1	B_2	B_3	B_4
A_1	1	1	4	-2
A_2	0	-4	-3	2

Find the optimal mixed strategy for this game and verify that the average value of the game is $-\frac{2}{3}$