Ollscoil na hÉireann, Gaillimh GX 2161
National University of Ireland, Galway
Summer Examinations, 2003/2004

Exam Code(s)	1BA1;1BS1;1EL1;1ER1;1MR1;1PT1;1CS1.
Exam(s)	First Year Science and First Arts
Module Code(s)	MP102
Module(s)	Mathematical Physics
Paper No	2
Repeat Paper	Special Paper
External Examiner(s)	Professor Brian Straughan
Internal Examiner(s)	Dr. M. Ó Confhaola
	Dr. P.M. O'Leary.

Instructions:

All students should do at least TWO questions from Section B.
Candidates taking the **Denominated Computing Studies** degree programme should answer a total of *FIVE complete* questions.
OTHERWISE full marks for *SIX complete* questions.
You may assume throughout that the Earth's gravitational acceleration is $g = 10ms^{-2}$.

Duration	3 hrs
No. of Answer books	
Requirements	
Handout	
MCQ	
Statistical Tables	Yes - Log Tables
Graph paper	
Log Graph Paper	
Other Material	
No. of Pages	3
Department(s)	Mathematical Physics

SECTION A

1. A lift descends from rest with acceleration k ms^{-2}, and after reaching a speed of 10 m/s continues at constant speed for 10s. It is brought to rest with a deceleration of $2k$ ms^{-2}. If the total time taken decending was 25s,

   a. Find the value of k ;

   b. Find the distance travelled during the descent.

2. A block of mass 3kg rests on a horizontal table. It is attached by means of a light, inextensible string to a particle of mass 8 kg. The string passes over a smooth pulley at the edge of the table. There is a frictional force of 15N opposing the motion of the block. Find

   a. the acceleration of the system;

   b. the tension in the string;

   c. the resultant force on the pulley.

3. A ball is kicked with a velocity of 15 m/s at an angle of 30^o to the horizontal towards a wall which is 8 m away.

   a. How far up the wall does the ball hit ?

   b. What is the speed of the ball when it hits the wall ?

   c. In what direction is the ball moving when it hits the wall ?

4. A ship A is moving due east at 18 km/h and another ship B is moving in a direction N 120^o east at 10 km/h. B is 12 km due north of A and both ships continue with the same velocities. Find the time when the ships are closest to each other and the shortest distance between them.

5. A 100 litre tank contains 20% pollutant. At time $t = 0$, a solution containing 10% pollutant flows into the tank at a rate of 5 litres/minute. The well-stirred mixture is drained off at the same rate.

   a. Find how much pollutant there is in the tank after 10 minutes;

   b. How long until there is 15 litres of pollutant in the tank ?

   c. How much pollutant will be in the tank as $t \to \infty$?

6. Find the position of the centre of mass of a uniform solid hemisphere of radius a.

   Prove that the centre of mass of a uniform hemispherical shell, whose inner and outer radii are a and b, is at a distance

$$\frac{3}{8}\frac{(a+b)(a^2+b^2)}{a^2+ab+b^2}$$

   from the centre and deduce the position of the centre of mass of a thin hemispherical shell.

SECTION B

7. A car is travelling round a circular bend of radius 25 m. The coefficient of friction between the car and the road is 0.3. The car has a mass of 500 kg.

   a. What is the maximum safe speed for the car if

      i) the road is unbanked

      ii) the road is banked at 15^o to the horizontal ?

   b. What is the minimum possible speed of the car if the road is banked at 15^o?

8. a. Solve the following second order linear differential equation

$$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 3x = 0$$

   subject to the initial condition $x(0) = 1$, $dx/dt(0) = 0$. Comment on the nature of the solution found.

   b. Solve the following second order linear differential equation

$$\frac{d^2x}{dt^2} + 9x = \sin(3t)$$

   subject to the initial condition $x(0) = 0$, $dx/dt(0) = 0$. Comment on the nature of the solution found.

9. A body of mass 2000 kg is in an elliptical orbit around a planet of mass 2.5×10^{26} kg. The body's furthest distance from the centre of the planet is 200 000 km, and at this point it is travelling at 27 000 km/h.

   a. Find the closest distance of the body to the planet;

   b. its speed at this point;

   c. the time to complete one orbit.

   [You may assume that $\ell/r = 1 + e\cos\theta$, where $\ell = h^2/GM$ and $h = r^2 d\theta/dt$, using standard notation, $G = 6.67\times10^{-11}\text{Nm}^2\text{kg}^{-2}$]

10. A uniform cylinder, of mass 3 kg and radius 0.2 m, rolls without slipping down a rough plane inclined at 20^o to the horizontal.

    a. Find the acceleration of its centre of mass;

    b. Use energy methods to find the speed of the centre of mass after it has moved 4 m.

11. A uniform ladder, of weight W, rests with one end against a smooth vertical wall and the other on a smooth horizontal floor. Its lower end is attached by means of a light inextensible string to the junction of the wall and the floor. A man of weight W climbs the ladder. Show that as the man moves from a point one-sixth of the way up the ladder to a point five-sixths of the way up, the tension in the string doubles.