Ollscoil na hÉireann, Gaillimh GX 2164 - GD
National University of Ireland, Galway
Summer Examinations, 2003/2004

Exam Code(s)	1BE1;1BN1;1BP1;1BI1;1BG1; 1BM1;1BP1;1BV1 IEG1;1BJ1.
Exam(s)	First Engineering
Module Code(s)	MP150
Module(s)	Mathematical Physics
Paper No	1
Repeat Paper	Special Paper
External Examiner(s)	Professor Brian Straughan;
Internal Examiner(s)	Dr. M. Ó Confhaola; Mr. S. MacConaonaigh.
Instructions:	Full marks will be awarded for SIX questions.
Duration	3 hrs
No. of Answer books	
Requirements	
Handout	
MCQ	
Statistical Tables	Yes - Log Tables
Graph paper	
Log Graph Paper	
Other Material	
No. of Pages	Total 3
Department(s)	Mathematical Physics

1. a) Find the equation of the line through the points with coordinates $(1,1,0)$ and $(2,1,1)$.

   Find the equation of the plane through these points and through the origin $(0,0,0)$.

   (b) The line of action of $\mathbf{F} = \mathbf{i} - \mathbf{j} + \mathbf{k}$ passes through the point $(1,0,1)$.

   (i) Find the moment of $\mathbf{F}$ about the origin.

   (ii) Find the moment of $\mathbf{F}$ about the point A at $(0,1,1)$.

   (iii) Find the moment of $\mathbf{F}$ about the line L through A in the direction of $\mathbf{k}$.

2. A car travels a distance of $500m$ in a straight line. It goes from rest with constant acceleration a_1 for 5 seconds at an average speed of $10ms^{-1}$. It then moves with constant speed and finally comes to rest with constant retardation a_2 for the last 10 seconds of its motion. Determine the total time for the journey.

3. Two particles A and B have velocities $(2\mathbf{i} + 3\mathbf{j})m/s$ and $(v(\mathbf{i} - \mathbf{j}))m/s$ respectively. Find $\mathbf{r}_{B|A}$, the position of B relative to A for all t given that $\mathbf{r}_{B|A} = (-2\mathbf{i} + 4\mathbf{j})m$ at $t = 0$ and find the value of v such that A and B collide. If $v = 6m/s$, find the time when A and B are closest together.

4. A light inextensible string passes over a smooth fixed pulley and carries at its ends a particle of mass $7kg$ and another smooth pulley, A, of mass $4kg$. A second string carrying $1kg$ and $3kg$ masses at its ends, hangs round A. Show that A does not move when the system is released.

5. A stands on a cliff of height $3h$ and B on the shore at a distance $2h$ from the foot of the cliff. Simultaneously, A throws a stone away from the cliff with speed u and angle of projection α and B throws a stone with speed v and angle of projection β away from the cliff where both trajectories are in the same vertical plane. If the stones collide show that

$$v \sin(\beta + \varepsilon) = u \sin(\alpha + \varepsilon)$$

   where $\tan \varepsilon = \frac{3}{2}$.

6. (a) Find the composite spring stiffness constant k when two springs with spring constants k_1 and k_2 are connected (i) in series (ii) in parallel.

   (b) A $1.5kg$ block is attached to two springs (i) in series and (ii) in parallel. The springs have spring constants $k_1 = 6N/m$ and $k_2 = 2N/m$. Find the period and frequency of the vertical oscillations in each case.

7. A particle of mass $20kg$ lies on a rough plane inclined at $45°$ to the horizontal. The coefficient of friction between the mass and the plane is $\mu = 1 - \frac{1}{2\sqrt{2}}$. The particle is attached, by means of a light inelastic string passing over a smooth pulley, to a freely hanging mass of $3kg$. The system is released from rest and the $20kg$ mass moves down the plane. Find its speed after it has moved a distance $d = 1.15m$. Take $g = 9.8m/s^2$.

8. Three smooth spheres, L, M and N, of equal volumes and of masses $1kg$, $3kg$ and $9kg$, respectively, are at rest on a smooth horizontal table. Their centres are collinear, with M lying between L and N. L is projected towards M, with speed $12m/s$. L hits M directly and, in turn, M hits N directly. If $e = \frac{1}{3}$, show that after the second impact both L and M are at rest. Find also the fractional loss of kinetic energy caused by the two collisions.

9. A particle describes vertical circles at the end of a light inelastic string $2m$ long. The ratio between the maximum and minimum tensions is 3. Find the speed of the particle at its highest point.

10. (a) Show that the centre of mass of a hemispherical solid of radius a is located on the axis of symmetry at a distance of $3a/8$ from the centre of the base.

    (b) A hemispherical bowl has internal radius $2a$ and external radius $3a$. Determine the position of its centre of mass.

11. A uniform ladder rests with one end against a smooth vertical wall and the other end on a rough horizontal floor, coefficient of friction $5\sqrt{3}/18$. The inclination of the ladder to the wall is 30 degrees. Show that a person whose mass is twice that of the ladder can just ascend to the top without the ladder slipping.

12. Find the moment of inertia of a solid sphere of mass M, radius a, about an axis through its centre.

    Hence, or otherwise, find the moment of inertia of a uniform hollow sphere of density ρ, internal radius a, external radius $2a$ about an axis through its centre.