Ollscoil na hÉireann, Gaillimh GX 2165
National University of Ireland, Galway
Summer Examinations, 2003/2004

Exam Code(s)	1FM1
Exam(s)	First Year Financial Mathematics and Economics
Module Code(s)	MP191
Module(s)	Mathematical Methods I
Paper No	1
Repeat Paper	
Special Paper	
External Examiner(s)	Professor Brian Straughan
Internal Examiner(s)	Dr. M. Ó Confhaola
	Dr. F. O'Dea

Instructions:

Answer THREE questions, at least ONE from each section.
All questions carry equal marks.

Duration	2 hrs
No. of Answer books	
Requirements	
Handout	
MCQ	
Statistical Tables	Yes - Log Tables
Graph paper	
Log Graph Paper	
Other Material	
No. of Pages	3
Department(s)	Mathematical Physics

SECTION A

1. Consider the first-order linear differential equation

$$x_{n+1} = Ax_n + B$$

where A and B are constants. Show that the solution can be expressed in the form

$$x_n = A^n x_0 + \frac{B(A^n - 1)}{(A-1)}, \quad A \neq 1,$$
$$x_n = x_0 + nB, \quad A = 1.$$

Use this formula to calculate the monthly repayments on a loan of €80,000 to be paid back over 20 years at an interest rate of 5%, compounded annually.

2. (a) Solve the difference equation

$$x_{n+2} - 8x_{n+1} + 15x_n = 16$$

subject to the conditions

$$x_0 = 1, \quad x_1 = 1$$

(b) Determine the general solution of the difference equation

$$x_{n+2} + 6x_{n+1} + 8x_n = 15n - 7$$

3. Let x denote the price of an object and $S(x)$ and $D(x)$ denote the supply and demand functions for that product. It is assumed that

$$S = a(x-c) + d$$
$$D = -b(x-c) + d,$$

for constants a, b, c, d. If it is further assumed that

$$D(x_{n+1}) = S(x_n)$$

show that we obtain

$$bx_{n+1} + ax_n = (a+b)c.$$

In the case where $a = 1$, $b = 3$, $c = 1$, $x_0 = 1$, find the values of x_n for $n = 1,2,3$ and predict the long-term solution.

SECTION B

4. A Building Society advertises that interest is compounded continuously at a rate of 6% per annum, so that *S(t)*, the amount compounded after a time t (measured in years), satisfies the differential equation

$$\frac{dS}{dt} = 0.06S(t)$$

(a) Integrate the above LODE and solve for *S(t)*, the compounded amount.

If initially €80,000 is borrowed on such an account, how long does it take to reach €150,000?

(b) Show that the solution to the differential equation

$$\frac{dy}{dx} = (\frac{3}{x+1} + 2x + 4)$$

subject to $y\,(0) = 4$, is given by

$$y(x) = \ln[(x+1)^3] + (x+2)^2$$

5. Of a group of 500 college students, 10 possess a mobile phone with the latest innovation. If the number N(t) that own an upgraded phone at time t (measured in months) satisfies the differential equation

$$\frac{dN}{dt} = 0.004N(500 - N).$$

Integrate the above NODE, and solve for *N(t)*, the number of students to own a phone with the innovation.

(a) Hence find the number of students that can be expected to own an upgraded phone after 2 months.

(b) Determine the long term behaviour of *N(t)*.