Ollscoil na hÉireann, Gaillimh
National University of Ireland, Galway

GX 2167

Semester II Examinations, 2003/2004

Exam Code(s)	2BS1, 2CA1, 2EL1, 2ER1, 2MR1, 2PT1, 3BS3, 3BS9, 3CS1, 3EL1, 3EL2, 3ER3, 4BS3.
Exam(s)	**Second Science and Arts**
Module Code(s)	MP206
Module(s)	Methods of Mathematical Physics(Pass)
Paper No.	
Repeat Paper	
Special Paper	
External Examiner(s)	Professor B. Straughan;
Internal Examiner(s)	Dr. M. S. Ó Confhaola; Dr. B. Gleeson.
Instructions:	Attempt *THREE* questions.
Duration	*TWO* hours
No. of Answer books	
Requirements:	
Handout	
MCQ	
Statistical Tables	YES, LOG TABLES
Graph Paper	
Log Graph Paper	
Other Material	
No. of Pages	2 PAGES (Excluding Cover Page)
Department(s)	MATHEMATICAL PHYSICS

1. (a) Find the derivative, with respect to distance, of the function

$$\phi(x,y,z) = 3x^2z + 3y - z^3,$$

along the parametric curve

$$\mathbf{r}(u) = (u-1, u^2-1, 2-2u),$$

at the origin $(0,0,0)$.

(b) Prove that the point $(2,-1,2)$ lies on each of the surfaces

$$x^2 - 2y^2 + 2z^2 = 10 \quad \text{AND} \quad z = x^2 + y^2 - 3.$$

Prove that these surfaces intersect at right angles at this point.

2. (a) Consider the integral

$$\int_0^3 \int_{x^2}^9 dy dx.$$

Sketch the region of integration A. Change the order of integration and hence evaluate the area A.

(b) Use plane polar coordinates to evaluate

$$\iint_{\mathrm{A}} 2(x^2 - y^2)\, dx dy$$

where A is the portion of the disc $x^2 + y^2 \leqslant 4$, $x, y \geqslant 0$.

(c) Consider the double integral

$$\iint_{\mathrm{A}} (x+y)\, d\,\mathrm{A},$$

where A is the *parallelogram* bounded by the lines $x+y=1$, $x+y=3$, $2x-y=0$, $2x-y=4$. By making the change of variable $u=x+y$, $v=2x-y$, and hence showing that the region of integration is a rectangle on the uv plane, evaluate the above integral.

3. Evaluate the line integral

$$\int_{(0,0,0)}^{(1,2,-3)} \mathbf{F} \cdot \mathbf{dr},$$

for the vector field

$$\mathbf{F} = (2xy^2 - yz)\,\mathbf{i} + (2x^2y - xz)\,\mathbf{j} - xy\mathbf{k}$$

along each of the following paths:

(a) the straight line joining $(0,0,0)$ directly to $(1,2,-3)$;

(b) the successive straight line segments from $(0,0,0)$ to $(0,2,0)$, from $(0,2,0)$ to $(0,2,-3)$ and from $(0,2,-3)$ to $(1,2,-3)$, in this order.

Prove that the vector field $\mathbf{F}$ is conservative.

4. The divergence theorem of Gauss can be written in the form

$$\iiint_{\mathrm{V}} \boldsymbol{\nabla} \cdot \mathbf{A} \, d\mathrm{V} = \iint_{\mathrm{S}} \mathbf{A} \cdot \hat{\mathbf{n}} \, d\mathrm{S}.$$

Explain what is meant by V, S and $\hat{\mathbf{n}}$ in this equation. Verify the divergence theorem for the vector field

$$\mathbf{A} = x^2 y \mathbf{i} - x y^2 \mathbf{j} + z \mathbf{k},$$

for the cylindrical region $x^2 + y^2 \leqslant 16$, $0 \leqslant z \leqslant 3$.

5. Find all the solutions of the equation

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, \quad 0 < x < L, \quad t > 0,$$

subject to the boundary conditions

$$u_x(0,t) = u_x(L,t) = 0, \quad t > 0,$$

and the initial condition

$$u(x,0) = f(x), \quad 0 < x < L.$$