

Ollscoil na hÉireann, Gaillimh
National University of Ireland, Galway

GX 2170

Semester II Examinations, 2003/2004

Exam Code(s) 2BS1.

Exam(s) Dámh na h-Éolaíochta agus Dámh na nDán

Module Code(s) MP208

Module(s) Modhanna na Fisice Matamaticíúla(Ónóracha)

Paper No.

Repeat Paper

Special Paper Tré Ghaeilge

External Examiner(s) Professor B. Straughan;

Internal Examiner(s) Dr. M. S. Ó Conghaola;

Dr. B. Gleeson;

Dr. F. Ó Dea.

Instructions: Freagair *TRÍ* cheist, ceann *AMHÁIN* ar a laighead as chaon roinn.

Duration *DHÁ* uair a chloig

No. of Answer books _____

Requirements:

Handout _____

MCQ _____

Statistical Tables YES, LOG TABLES

Graph Paper _____

Log Graph Paper _____

Other Material _____

No. of Pages 3 PAGES (~~Excluding Cover Page~~)

Department(s) MATHEMATICAL PHYSICS

- c. na mírlínte díreach a théann uaidh $(0,0,0)$ go $(0,2,0)$, uaidh $(0,2,0)$ go $(0,2,-3)$ agus uaidh $(0,2,-3)$ go $(1,2,-3)$, ins an ord seo.

Cruthaigh go bhfuil an raonna veicteoiriúil \mathbf{F} caomhnach. Faigh tré suimeáil i leith líne no ar bhealach ar bith eile, an fheidhm scálach tualainghe $\phi(x,y,z)$ i gcóir an raonna seo \mathbf{F} .

Roinn B

4. Is féidir teóragán díbhéirseach Gauss a scríobh san bhfuirm

$$\iiint_V \nabla \cdot \mathbf{A} dV = \iint_S \mathbf{A} \cdot \hat{\mathbf{n}} dS.$$

Mínigh céard is ciall le V , S agus $\hat{\mathbf{n}}$ san gcothromóid seo. Fíoraigh an teóragán díbhéirseach i gcóir an raonna veicteoiriúil

$$\mathbf{A} = x^2 y \mathbf{i} - xy^2 \mathbf{j} - 2xz^2 \mathbf{k},$$

i gcóir an réigiún sorchóireach $x^2 + y^2 \leq 16$, $0 \leq z \leq 3$.

5. Spherical Polar co-ordinates (r, θ, ϕ) are related rectangular Cartesian co-ordinates (x, y, z) by the relationships:

$$x = r \sin(\theta) \cos(\phi), \quad y = r \sin(\theta) \sin(\phi), \quad z = r \cos(\theta).$$

- Calculate the scale factors h_r , h_θ and h_ϕ and verify that the system is orthogonal.
- Find $\nabla \Psi$ in terms of this co-ordinates system.
- Find $\nabla^2 \Psi$ in terms of these co-ordinates.

[You may assume that, in general, for an orthogonal system

$$\nabla^2 \Psi = \frac{1}{h_1 h_2 h_3} \left\{ \frac{\partial}{\partial u_1} \left(\frac{h_2 h_3}{h_1} \frac{\partial \Psi}{\partial u_1} \right) + \frac{\partial}{\partial u_2} \left(\frac{h_3 h_1}{h_2} \frac{\partial \Psi}{\partial u_2} \right) + \frac{\partial}{\partial u_3} \left(\frac{h_1 h_2}{h_3} \frac{\partial \Psi}{\partial u_3} \right) \right\} \Bigg].$$

- 6.
- Define Cartesian tensors of ranks one and two.
 - Determine the rotation matrix associated with a rotation of Cartesian frame about its x_3 axis, the rotation being of angle α in an anti-clockwise direction. If the new frame is rotated by an angle β about the new x_2 axis, calculate the rotation matrix for the composite rotation.
 - A tensor of rank two has components

$$T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

with respect to a given Cartesian frame. The frame is rotated by an angle α about its x_3 axis (see part (b) of this question). Find the components of the tensor in the rotated frame.

Roinn A

1.

- a. Faigh díorthach na feidhme i leith an fhaid

$$\phi(x, y, z) = 3x^2z + 3y - z^3,$$

i leith an chuair

$$\mathbf{r}(u) = (u - 1, u^2 - 1, 2 - 2u),$$

ag an mbunphointe $(0, 0, 0)$.

- b. Cruthaigh go luíonn an pointe $(2, -1, 2)$ ar chaon cheann de dá dhromachla

$$x^2 - 2y^2 + 2z^2 = 10 \quad \text{agus} \quad z = x^2 + y^2 - 3.$$

Cruthaigh go ngearann an dá dhromachla a chéile go h-ingearach ag an bpointe seo.

- c. Fíoraigh an ionnas veicteoiriúil

$$\text{curl}(\text{curl } \mathbf{F}) = \text{grad}(\text{div } \mathbf{F}) - \nabla^2 \mathbf{F}$$

i gcóir an raonna veicteoiriúil $\mathbf{F}(x, y, z) = 2x\mathbf{i} + 3y\mathbf{j} + 4z\mathbf{k}$.

2.

- a. Maidir leis an suimeálaí

$$\int_{-3}^3 \int_{x^2}^9 dy dx.$$

déan sceitse den réigiún suimeála A. Athraigh ord na suimeála agus faigh an achar A.

- b. Bain feidhm as comh ordanáidí polúla chun luacháil a dhéanamh ar

$$\iint_A 2(x^2 - y^2) dx dy$$

áit gurbh é A an chuid den diosca $x^2 + y^2 \leq 4$, $x, y \geq 0$.

- c. Déan luacháil ar an suimeálaí

$$\iint_A (x + y) dA,$$

áit gurbh é A an *comhthreórmhán* cuimsithe ag na línte $x + y = 1$, $x + y = 3$, $2x - y = 0$, $2x - y = 4$. Déan an athrú athróige $u = x + y$, $v = 2x - y$.

3. Déan luacháil ar an suimeálaí

$$\int_{(0,0,0)}^{(1,2,-3)} \mathbf{F} \cdot d\mathbf{r},$$

i gcóir an raonna veicteoiriúil

$$\mathbf{F} = (2xy^2 - yz)\mathbf{i} + (2x^2y - xz)\mathbf{j} - xy\mathbf{k}$$

i leith gach ceann de na conair seo leannas:

- a. an líne díreach a cheanglaíonn $(0, 0, 0)$ le $(1, 2, -3)$;
b. an cuar paraiméadrach $\mathbf{r}(u) = (u, 2u^2, -3u^3)$