

Ollscoil na hÉireann, Gaillimh
National University of Ireland, Galway
Semester II Examinations, 2004/2005

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| Exam Code(s) | 1MS1, 1SD1 |
| Exam(s) | MS1 M.Sc.Degree |
| | SD1 H.Dip.(Software Design And Development) |
| Module Code(s) | CT853 |
| Module(s) | Algorithmics & Logical Methods |
| Paper No. | 1 |
| Repeat Paper | Special Paper |
| External Examiner(s) | Professor D. Bell |
| Internal Examiner(s) | Professor G. Lyons |
| | Dr. M. Mc Gettrick |

Instructions

Answer 3 questions.
 All questions will be marked equally.

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|---------------------|------|
| Duration | 2hrs |
| No. of Answer Books | 1 |

Requirements

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| Handout | |
| MCQ | |
| Statistical Tables | |
| Graph Paper | |
| Log Graph Paper | |
| Other Material | |

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| No. of Pages | |
| Department(s) | |

1. (a) Using each of the following methods, write down (step by step) the position of each letter in the word "harlem" when sorted using
 - (i) selection sort
 - (ii) merge sort
- (b) State (in "Big Oh" notation) the worst case complexity of both algorithms in part (a). For merge sort, explain why you would expect a factor of approximately 2 between worst case and best case performance.

2. (a) Write the pseudocode for an algorithm which calculates x^n , given as input x and n where n is a positive integer, using
 - (i) Iteration
 - (ii) Recursion
- (b) Write the pseudocode for a recursive algorithm which checks whether or not a word is a palindrome (a palindrome is a word which reads the same written in either direction, e.g. "deed", "abba").

3. Consider the following two algorithms:

```
(I)
BEGIN {algorithm to calculate  $x^n$ }
IN( $x, n$ )
 $y \leftarrow 1$ 
WHILE ( $n > 0$ ) DO
  IF ( $n \bmod 2 = 0$ ) THEN
    BEGIN  $x \leftarrow x * x$ ;  $n \leftarrow n/2$ ; END
  ELSE
    BEGIN  $y \leftarrow y * x$ ;  $n \leftarrow n-1$ ; END
OUT( $y$ )
END
```

```
(II)
BEGIN {algorithm to calculate  $x^n$ }
IN( $x, n$ )
OUT(POW( $x, n$ ))
END

POW( $i, j$ )
IF ( $j=1$ ) THEN POW  $\leftarrow i$ 
ELSE POW  $\leftarrow$  POW( $i, \text{FLOOR}(j/2)$ ) *
  POW( $i, \text{FLOOR}((j+1)/2)$ )
```

(note that $p \bmod q$ returns the remainder after dividing p by q , e.g. $7 \bmod 2$ is 1)

- (a) For large n , which algorithm should be faster?
- (b) For algorithm (I), determine the number of times the while loop is executed for the two inputs $n = 64$ and $n = 63$. Hence calculate the worst case and best case complexity of algorithm (I).
- (c) Say that for certain fixed values of x and n both algorithms run in equal time t_1 seconds. Estimate (in terms of t_1) the time for each algorithm if we keep x constant but double n .

4. (a) Use truth tables to determine whether each of the following well formed formulae (wff) are tautologies, contradictions, or neither.

(i) $\neg(A \vee B) \leftrightarrow \neg A \wedge \neg B$

(ii) $A \vee (B \wedge C) \leftrightarrow (A \vee B) \wedge (A \vee C)$

(iii) $\neg(A \wedge B) \rightarrow (A \vee \neg B)$

- (b) Given the predicates

$E(x)$: "x is an Eagle"

$M(x)$: "x is a Mouse"

$A(x,y)$: "x Attacks y"

write statements in First Order Predicate Calculus to express each of the following.

- (i) All Eagles Attack Mice.
(ii) Some Eagles Attack all Mice.
(iii) Only Eagles Attack Mice.