914

Ollscoil na hÉireann, Gaillimh
National University of Ireland, Galway

Semester II Examinations, 2004/2005

Exam Code(s)	2BA1, 2BA5, 2BC1, 2BC2, 2BC3, 2BC4, 2BC5, 2FM1, 1OA1, 1EM1
Exam(s)	2nd Arts, 2nd Commerce, 2nd Commerce (Language) 2nd Financial Mathematics and Economics
Module Code(s)	EC 218
Module(s)	Mathematical Economics
Paper No.	1
Repeat Paper	Special Paper
External Examiner(s)	Professor Vincent Munley
Internal Examiner(s)	Mr. Brendan Kennelly Dr. Gerald Pech
Instructions:	Answer any THREE of the four questions.
Duration	2 hours
No. of Answer books	
Requirements:	1
Handout	
MCQ	
Statistical Tables	
Graph Paper	
Log Graph Paper	
Other Material	
No. of Pages	3
Department(s)	Economics

Examination in Mathematical Economics (EC218)

Answer any THREE of the following four questions.

1. (Quadratic optimization)

A firm produces two widgets, x and y. The firm perceives the demand curves for either widget to be downward sloping and the prices P_x and P_y to depend on the quantity sold:

$$P_x = 3 - \frac{1}{2}x$$

$$P_y = 6 - \frac{1}{2}y.$$

Either widget uses up capacity of both of the two machines which the firm owns. The constraints for machine capacities are

machine I: $6 \geq x + y$
machine II: $9 \geq 2x + y$.

The objective of the firm is to maximize profits with its given production facilities.

a) Determine the profit function. Show that the isoprofit contours are circles.

b) Plot the constraints into a graph and mark the feasible set. Locate the unconstrained optimum in the graph.

c) Analytically solve the constrained optimization problem (hint: use the information from the graph in part b) in order to minimize the steps to find the solution). Why is the found solution the unique constrained optimum?

d) The production engineer says that he could improve the efficiency of the production process such that the factor requirement of product x of machine I would be reduced. The manager objects that this would be pointless because the constraint for machine II would become binding. Is the manager right? Try to locate the critical point where constraint II becomes binding in the graph. What conditions must this point fulfill? Do not try to analytically solve this part of the problem.

2. (Linear programming)

Indiana Jones has discovered the treasure of the Pharaoe consisting of practically unlimited amounts of gold coins and artefacts. Unfortunately, the pyramid which contains the treasure threatens to collapse in 60 minutes. He has only a limited capacity in his bag and artefacts are more bulky. Therefore, he reckons that on average it takes him 1 minute to get one kilo of gold coins and 2 minutes to get one kilo of artefacts out of the pyramid. The run down biplane with which he intends to escape from the scene can only carry Indiana Jones himself plus a maximum additional weight of 50 kilos. Because his enemies will arrive shortly after the collapse of the pyramid, there is no question of coming back. The value of gold coins on the black market in Cairo is 500 $ per kilo gold coins and one kilo of artefacts is expected to generate 800 $. If Indiana Jones could solve this planning problem in no time and he wants to maximize the revenue from selling his loot, what does he carry with him when he arrives in Cairo?

a) Set up the optimization problem.

b) Graphically, determine the optimal solution.

c) Show that the first-order and Kuhn-Tucker conditions hold for the solution found in b).

d) Set up the dual of the optimization problem in part a) and sketch the dual problem in a graph. How do you interpret the objective function and the constraints of the dual?

3. (Expected utility theory)

Simple Simon's von Neumann-Morgenstern utility function is $u(y) = \sqrt{y}$ where y is his income. He is taking 1000 eggs to the market to be sold at a net profit of 10 cent each. The eggs are his only source of income and there is 50% chance that in any one journey he will break all the eggs he's carrying.

a) Assuming he has time for two journeys before the market opens. Will he take his eggs to the market in one or two journeys?

b) Assuming he has overslept so that there is only time for one journey but Dr Foster (who is as clumsy as he is and whose probability of egg breaking is also 50%) offers to help him out? What is the maximum amount he would be prepared to pay Dr Foster to carry 500 eggs to the market? Which of the following choices is the right answer (no marks without showing the method)?

(1) € 10.35 (2) € 10.7 (3) € 7.28 (4) nothing

please turn over

c) During the following week an insurance agent pays a visit to Simon's farm and offers him insurance. Simon can buy cover C for his load of eggs at a premium of 0.6 C. What is the optimal amount of cover which Simon wants to take out?

d) Will Simon sign the contract if he believes that he can hire Dr Foster at the price calculated in part b) the following weekend and why? (No calculation necessary here).

4. (Game theory/duopoly)

There are two firms i = 1, 2 competing in a duopolistic market with a homogenous product. Output levels are Q_1 and Q_2 respectively and total provision of the market is $Q = Q_1 + Q_2$. The market price is given by the market demand curve

$$P = 120 - \frac{1}{2} Q$$

where Q is total provision of the market. Marginal cost is 30 for either firm. The firms' strategic variable is their output level Q_i.

a) Determine the reaction correspondence for each firm and draw the two reaction curves into one graph.

b) Analytically solve for the Cournot-Nash equilibrium and determine market supply, prices and profits.

c) Develop an argument from the graph in part a) to show that the Cournot-Nash equilibrium is not a Pareto-optimum.

d) Write down the optimization problem of firm 1 if it takes the role of a Stackelberg leader. How do you expect the outcome to differ from outcome b) and why?