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Ollscoil na hÉireann, Gaillimh
National University of Ireland, Galway

Semester II Examinations 2004 / 2005

Exam Code(s)	4BN121 4BP121
Exam(s)	BE Degree in Electronic Engineering BE Degree in Electronic and Computer Engineering
Module Code(s)	EE409
Module(s)	Digital Signal Processing
Paper No. Repeat Paper	1
External Examiner(s) Internal Examiner(s)	Prof. S. McLaughlin Prof. D.J. Wilcox Dr. E. Jones

Instructions: Answer five questions
All questions carry equal marks

Duration	3 hrs
No. of Pages	(including cover page)
Department(s)	Electronic Engineering
Course Co-ordinator(s)	

Requirements:

MCQ	
Handout	
Statistical Tables	
Graph Paper	
Log Graph Paper	
Other Material	Standard mathematical tables

1. (a) Determine the difference equation of the discrete-time system shown in Figure 1. Hence, determine the first five samples of the system impulse response. **[8 marks]**.

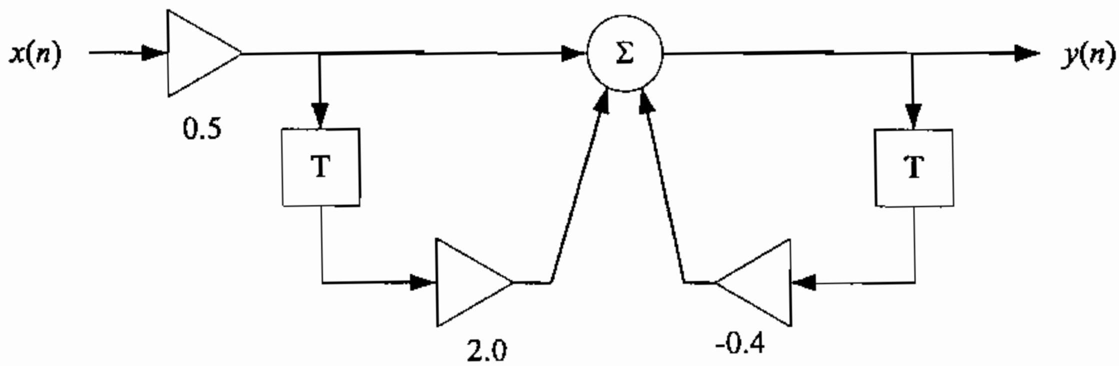

Figure 1.

(b) Sketch the following impulse responses, and state if the digital filters to which they correspond are (i) causal or non-causal, (ii) stable or unstable:

(*i*) $h_1(n) = \left(\frac{2}{3}\right)^n u(n)$

(*ii*) $h_2(n) = \left(\frac{5}{4}\right)^n u(n)$

(*iii*) $h_3(n) = \left(\frac{1}{3}\right)^{-n} u(-n)$

(*iv*) $h_4(n) = \left(\frac{3}{2}\right)^{-n} u(-n)$

[4 marks]

(c) A system for sampling a continuous-time audio signal consists of an analogue to digital converter preceded by an anti-aliasing Butterworth filter that has a cut off frequency of 18 kHz. The sampling frequency is 44.1 kHz. In order to minimise aliasing, the magnitude response of the anti-aliasing filter must be –30 dB or lower at the Nyquist frequency. Calculate the order of the Butterworth filter required to minimize aliasing. Suggest a method by which the complexity of the anti-aliasing filter could be significantly reduced. **[8 marks]**.

Note that the order of a Butterworth filter, n, is given by the following expression:

$$n = \frac{\log_{10}\left[\dfrac{10^{-\frac{K_1}{10}} - 1}{10^{-\frac{K_2}{10}} - 1}\right]}{2\log_{10}\left(\dfrac{f_1}{f_2}\right)}$$

where the gain of the filter is equal to K_1 dB at a frequency f_1 Hz, and is equal to K_2 dB at a frequency f_2 Hz.

2. (a) A discrete-time system has a finite-duration impulse response that consists of the samples {2, 1, 2}, commencing at $n = 0$. Using convolution, calculate the response of the system to a finite-duration input signal that consists of the samples {1, 3, 4, 1}, commencing at $n = 0$. In particular, indicate *in detail* (using graphs where appropriate) the calculations needed to determine $y(2)$. **[7 marks]**.

[cont'd]

(b) A discrete-time system has a complex conjugate pole pair at $z = 0.8 + j0.5$ and $z = 0.8 - j0.5$, as well as a double zero at $z = 0$. Based on the pole locations, write down an equation for the general form of the impulse response as a function of n, the sample index, and sketch the impulse response (you do not need to calculate individual sample values). Also, determine the transfer function and hence obtain an expression for the difference equation. **[7 marks].**

(c) Using the z-Transform convolution property, determine the sequence resulting from the convolution of the following pair of sequences :

$x(n) = \{1, 2, 3, 1, -1\}$, commencing at $n = 0$, and $y(n) = \{1,3, 1\}$, commencing at $n = 0$.

[6 marks].

3. (a) The transfer function of a simple first-order low-pass filter is described by the following equation:

$$H(z) = \frac{0.08}{1 - 0.92z^{-1}}$$

The sampling rate is 20 kHz. Determine the cut off frequency of the filter in Hz (hint: obtain an expression for the squared magnitude response of the filter). **[5 marks].**

(b) A digital filter contains a zero at $z = 0.8$, as well as poles at $z = -0.5 + j0.3$ and $z = -0.5 - j0.3$. Sketch the pole-zero map of the filter. Hence, calculate the magnitude response of the filter at a frequency of 100 Hz, if the sampling frequency is equal to 800 Hz. **[5 marks].**

(c) Using the pole-zero placement method, obtain the transfer function of a digital resonator with the following characteristics:

(i) Sampling rate of 8 kHz
(ii) Centre frequency of 1 kHz
(iii) Bandwidth of 60 Hz
(iv) DC gain of 1

Sketch the pole zero map of the filter, and give an expression for the difference equation. **[6 marks].**

(d) A signal processing application requires the calculation of the spectrum of a signal that has a sampling frequency of 16 kHz. The spectral analysis is to be carried out using short-time analysis with a window length of no more than 25 msec, with a resolution such that the frequency step is no greater than 20 Hz. Calculate the minimum number of samples that must be used to zero-pad each frame of the signal in order to achieve the desired frequency resolution. You may assume that efficient FFT algorithm is used for spectral analysis (hint: this will place restrictions on the number of samples input to the algorithm). **[4 marks].**

4. (a) A biomedical signal processing application requires removal of mains hum (50 Hz) from an electrocardiogram (ECG) signal that is sampled with a frequency of 500 Hz. Determine the transfer function of a notch filter suitable for this purpose. You may assume that a notch of width 20 Hz will suffice. **[6 marks].**

(b) Using the Impulse Invariant Transformation, design a digital filter based on the following continuous-time transfer function:

$$H(s) = \frac{4}{(s+2)(s+4)}$$

Assuming that the sampling rate is chosen to be ten times the highest pole frequency in the analogue filter, calculate the digital filter coefficients. **[7 marks].**

(c) A simple first-order analogue filter is described by the following transfer function:

$$H(s) = \frac{\omega_c}{s + \omega_c}$$

where ω_c is the cut off frequency in radians/s.
Using the bilinear transformation, determine the transfer function of the digital equivalent of this filter, if the desired cutoff frequency is 20 kHz and the sampling frequency is 128 kHz.
If pre-warping was not carried out, what would be the actual cut off frequency of the digital filter? **[7 marks].**

5. (a) The input signal to the system in Figure 2 is a band-pass signal whose spectrum extends from 250 Hz to 500 Hz. Following upsampling by a factor of 2, the signal is multiplied by a carrier with a frequency of 2 kHz, thus generating a double sideband, suppressed carrier signal $r(n)$. The filter $G(\theta_p)$ is a low-pass filter.

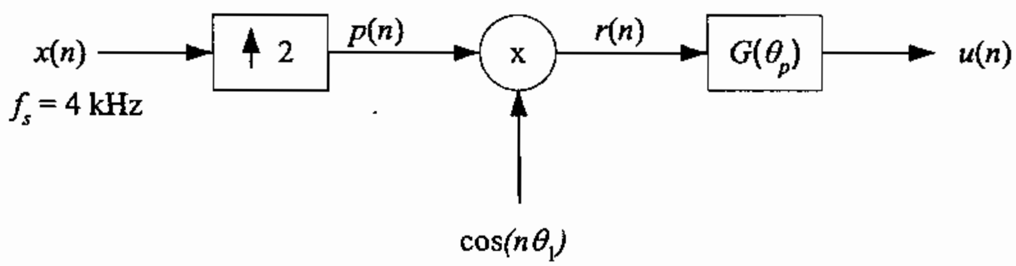

Figure 2.

(i) Determine the sampling frequency of each of the signals in Figure 2.
(ii) Sketch the magnitude spectrum of the input signal, $|X(\theta)|$, as a function of frequency, θ.
(iii) Sketch the magnitude spectra $|P(\theta_p)|$ and $|R(\theta_p)|$ against θ_p, where θ_p is the digital frequency variable for the upsampled signal.
(iv) Determine the minimum cutoff frequency in Hz of the low-pass filter, if the signal $u(n)$ is to contain only one sideband (assume the lowpass filter is ideal), and sketch the resulting magnitude spectrum $|U(\theta_p)|$ against θ_p.

[13 marks]

(b) Discuss how the sampling rate of a signal may be changed by a non-integer factor. In particular, indicate how the cut off frequencies of any filter used are chosen.
A signal with a sampling frequency of 10 kHz must be re-sampled to 8 kHz. Draw a block diagram of a system, using up-samplers, filters and down-samplers, to achieve this. Indicate clearly the cut-off frequency of any filters (you may assume that the filters are "ideal"). **[7 marks].**

6. (a) In the speech production model, a commonly used approximation for the impulse response of the glottal pulse-shaping filter is as follows:

$$g(n) = na^n, n \geq 0$$
$$g(n) = 0, n < 0$$

By obtaining the transfer function of this filter, choose a value of a such that the the magnitude response at DC is 40 dB higher than the response at half the sampling frequency. **[7 marks].**

(b) Outline the components of an algorithm that could be used for endpoint detection of speech in a computer speech recognition system. In particular, explain what speech analysis techniques would be used. Use diagrams and equations where appropriate. **[5 marks].**

[cont'd]

(c) A segment of speech with a sampling rate of 8 kHz is filtered with a low-pass filter filter with cutoff frequency equal to 2 kHz, and is then analysed using Linear Predictive Coding (LPC) of order 4. The resulting fourth order all-pole vocal tract transfer function is given in factored form as follows:

$$H(z) = \frac{1}{(1 - 1.4966z^{-1} + 0.81z^{-2})(1 - 0.4326z^{-1} + 0.49z^{-2})}$$

Given the transfer function, estimate the two formant frequencies and their approximate bandwidths, and sketch the magnitude spectrum. **[8 marks].**

Note: Use the following approximate relationship between bandwidth and pole radius:

$$r \approx 1 - \frac{\Delta f}{f_{samp}}\pi$$

where r is pole radius, and Δf is approximate bandwidth.

7. (a) Using block diagrams, describe how an adaptive filter may be used in the following applications:
(i) echo cancellation in a digital modem
(ii) equalization in a digital modem

[6 marks]

(b) An adaptive echo canceller is required as part of a transceiver in a high-speed datacommunications system. The input data to the echo canceller is a random sequence of data symbols, with values taken from the following set [-7, -5, -3, -1, 1, 3, 5, 7]. All values are assumed equiprobable. The sampling rate of the system is 100 kHz, and the length of the echo path is approximately 2.7 milliseconds.

(i) Calculate the minimum number of taps needed in the echo canceller.
(ii) Assuming the Least Mean Squares (LMS) algorithm is to be used for adaptation, choose a suitable value for the adaptation step-size.
(iii) Discuss how the implementation of the echo canceller may be made more efficient, given the characteristics of the input data.

[8 marks].

(c) With the aid of graphs, discuss how the step size used in the LMS algorithm affects convergence speed and steady-state error. **[6 marks].**