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OLLSCOIL NA hÉIREANN, GAILLIMH
NATIONAL UNIVERSITY OF IRELAND, GALWAY

SUMMER EXAMINATIONS 2005

B.Sc. (Part I) and B.A EXAMINATION

MATHEMATICS
MA342 – TOPOLOGY
HONOURS

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Time allowed: **Two** hours.
Attempt **one** question from Section A, **one** question from Section B and **one** question from Section C.

Section A

1. (a) What does it mean to say that a space X has the fixed point property?
   (b) Show that the interval $(1, 2)$ does not have the fixed point property.
   (c) Suppose that X and Y are homeomorphic metric spaces. Show that X has the fixed point property if and only if Y has the fixed point property.

2. (a) What is a topological space?
   (b) Explain what is meant by the cofinite topology on a set X and show that it satisfies the axioms of a topology.
   (c) Suppose that X is a compact topological space, Y is a topological space and that $f : X \to Y$ is surjective continuous function. Show that Y is also compact.

Section B

3. (a) Let $D^2 = \{(x,y) \in \mathbb{R}^2 : x^2 + y^2 \leq 1\}$ and let $S^1 = \{(x,y) \in \mathbb{R}^2 : x^2 + y^2 = 1\}$. Show that statement A below is implied by statement B:

   A. Every continuous function $f : D^2 \to D^2$ has a fixed point.

   B. Given any continuous function $f : D^2 \to \mathbb{R}^2$ such that $f(x,y) = (x,y)$ whenever $(x,y) \in S^1$, there is some $(x_0, y_0) \in D^2$ such that $f(x_0, y_0) = (0,0)$.

   (b) Explain what is meant by a "*triangulation of a space X*".

   (c) State Sperner's Lemma (you are not required to give a proof).

4. (a) Let (x_n) be a bounded sequence in $\mathbb{R}$. Show that (x_n) has a convergent subsequence.

   (b) Which of the following subsets of $\mathbb{R}^2$, if any, are sequentially compact? Explain your answers.

   i. $\{(x,y) : x^2 + y^2 \leq 1\}$.
   ii. $\{(x,y) : 1 \leq x + y \leq 2\}$.
   iii. $\{(x,y) : x^2 + y^2 < 1\}$.

Section C

5. Explain the problem of rent division and explain how Sperner's Lemma may be used to show that, under certain conditions, the rent division problem has a solution.

6. Let $D^2 = \{z \in \mathbb{C} : |z| \leq 1\}$, let $f : D^2 \to \mathbb{C}$ be a continuous function and let n be a positive integer. Suppose that $f(z) = z^n$ for all z such that $|z| = 1$. Show that there is some $z_0 \in D^2$ such that $f(z_0) = 0$.