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OLLSCOIL NA hÉIREANN, GAILLIMH
NATIONAL UNIVERSITY OF IRELAND, GALWAY

SUMMER EXAMINATIONS 2005

1AH1 H. Diploma Arts (Mathematics)
1AH2 Master of Arts (Mathematics)

MA535 – TOPICS IN ANALYSIS II

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Time allowed: Two hours.

Attempt one question from Section A, one question from Section B and
one question from Section C.

Section A

1. (a) What does it mean to say that a space X has the fixed point property?
(b) Show that the interval $(1, 2)$ does not have the fixed point property.
(c) Suppose that X and Y are homeomorphic metric spaces. Show that X has the fixed point property if and only if Y has the fixed point property.
2. (a) What is a topological space?
(b) Explain what is meant by the cofinite topology on a set X and show that it satisfies the axioms of a topology.
(c) Suppose that X is a compact topological space, Y is a topological space and that $f : X \rightarrow Y$ is surjective continuous function. Show that Y is also compact.

Section B

3. (a) Let $D^2 = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 \leq 1\}$ and let $S^1 = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 = 1\}$. Show that statement A below is implied by statement B:
- A. Every continuous function $f : D^2 \rightarrow D^2$ has a fixed point.
 - B. Given any continuous function $f : D^2 \rightarrow \mathbb{R}^2$ such that $f(x, y) = (x, y)$ whenever $(x, y) \in S^1$, there is some $(x_0, y_0) \in D^2$ such that $f(x_0, y_0) = (0, 0)$.
- (b) Explain what is meant by a “*triangulation of a space X*”.
- (c) State Sperner’s Lemma (you are not required to give a proof).
4. (a) Let (x_n) be a bounded sequence in \mathbb{R} . Show that (x_n) has a convergent subsequence.
- (b) Which of the following subsets of \mathbb{R}^2 , if any, are sequentially compact? Explain your answers.
- i. $\{(x, y) : x^2 + y^2 \leq 1\}$.
 - ii. $\{(x, y) : 1 \leq x + y \leq 2\}$.
 - iii. $\{(x, y) : x^2 + y^2 < 1\}$.

Section C

5. Explain the problem of rent division and explain how Sperner’s Lemma may be used to show that, under certain conditions, the rent division problem has a solution.
6. Let $D^2 = \{z \in \mathbb{C} : |z| \leq 1\}$, let $f : D^2 \rightarrow \mathbb{C}$ be a continuous function and let n be a positive integer. Suppose that $f(z) = z^n$ for all z such that $|z| = 1$. Show that there is some $z_0 \in D^2$ such that $f(z_0) = 0$.